International Journal of Systems Science and Applied Mathematics
2022; 7(2): 39-45
http://www.sciencepublishinggroup.com/j/ijssam
doi: 10.11648/j.ijssam.20220702.12
ISSN: 2575-5838 (Print); ISSN: 2575-5803 (Online)

Markovian Approach for Analyzing Patient Flow Data: A Study of Kapsabet County Referral Hospital, Kenya

Merary Kipkogei[*], Edgar Ouko Otumba

Department of Statistics and Actuarial Sciences, Maseno University, Maseno, Kenya

Email address:
kipkogeimerary01@gmail.com (M. Kipkogei)
[*]Corresponding author

To cite this article:
Merary Kipkogei, Edgar Ouko Otumba. Markovian Approach for Analyzing Patient Flow Data: A Study of Kapsabet County Referral Hospital, Kenya. *International Journal of Systems Science and Applied Mathematics*. Vol. 7, No. 2, 2022, pp. 39-45.
doi: 10.11648/j.ijssam.20220702.12

Received: June 10, 2022; **Accepted**: June 30, 2022; **Published**: July 12, 2022

Abstract: Hospital is indispensable and necessary welfare of society. Through it, we can manage our illnesses by treatment and prevention interventions. With the rise incidences of chronic diseases and illnesses, there has been an increased demand for health care services round the world. This demand has subsequently caused a serious pressure resulting to serious episodes of congestion and overcrowding in hospitals. Hospital overcrowding and congestion, has always been a problem to patients, hospital administration and to the general health workers. Hospitals are struggling to alleviate congestion and overcrowding. In this study, we developed an objective patient flow estimation using Markov chain models. Weekly data from Kapsabet County Referral Hospital facility was used to assess the flow. Markov chains' transition probability matrices were constructed for each day in a week. Markov chain's four-state model used was; High, Medium, Low and Very Low. The future *n* step transition probabilities matrices were computed, giving rise to steady state for each day of the week. It was examined that the patient flow had some pattern through the Markov chains' steady states. The steady state probability of the flow is high on Mondays with highest probability of 0.57. Medium on Tuesdays through to Thursdays with steady state probabilities ranging from 0.36 and 0.3 respectively. On Fridays the probabilities decrease from 0.22 to 0.12 on Sunday. Through this study, we can witness some pattern from steady state of transition matrices. This way, the patient's population flow throughout the week at this facility is identified. Generally, through this study, the patient flow is understood and hence the patient flow congestion can be easily attenuated.

Keywords: Markov Chain Models, Patient Flow, Steady State, Transition Probability Matrix

1. Introduction

Hospital is an essential welfare of society. Hospital provides management of illnesses through treatment and prevention interventions. This is done by medical and health professionals to the general public [1]. Patient flow constitutes the prowess of health care to serve the patients quickly and efficiently as they move through the stages of care without any hiccup. Globally, there has been a continuous increase in chronic diseases and illnesses which has caused increased demand for health care services in hospitals [2]. Because of this, episodes of congestion and overcrowding tend to happen. This problem amounts to wide scope of ramifying consequences such as long queues, long waiting times, lowering attention from practitioners and crowded facilities [3]. Furthermore, hospital congestion and overcrowding may compromise the quality of care. Hospital overcrowding is a real problem to patients, hospital administrators and to the general health workers at large. It has continually persisted despite the awareness of the problem. Besides, it being has been a concern through the years. To maintain the quality of the services offered to the patients at the hospital, there is need to evaluate 'flow'. One of the ways of doing this is by analyzing the patient flow data. This paper purposes to analyze patient flow data at Kapsabet County Referral Hospital using Markov chain models.

1.1. Patient Flow

Patient flow can be defined as a study of investigating on ways in which patients are transferred inside the health care system [4]. Patient flow is also ensuring that the patients receive the care quickly and efficiently as they pass through the stages of care when they need it without minimal delay. A well-organized patient flow can reduce delay of health care services [5]. Patient flow analysis is a quality improvement tool that helps the healthcare facilities identify inefficiencies in patient flow and provide a suggestion on how to improve the patient flow process [6, 7]. The most important step is to identify the patient flow through a certain process [8].

1.2. Basic Concepts

This section presents the concepts and theories underpinning the study. This study was anchored on the theory of Markov Chains. A Markov chain is a special type of stochastic process. The concepts of stochastic process, Markov Chain, transition matrix, Properties of Markov Chains and Classification of States are presented in sub sections as outlined below.

1.2.1. Stochastic Process

A stochastic process is a random process, that is, a change in the state of some system over time whose course chance and for which the probability of a particular course is defined. It is basically a family of random variables, $x(t); t \in T$ defined on a given probability space, indexed by the time variable t, where t varies over an index set T. A stochastic process may be continuous or discrete. A stochastic process is said to be a discrete time process if set T is finite or countable that is, if $T = 0,1,2,3, \ldots, n$resulting in the time process $x(0), x(1), x(2), x(3), \ldots, x(n)$ is recorded at time $0,1,2,3, \ldots, n$ respectively. On the other hand, stochastic processes $x(t); t \in T$ considered a continuous time process if T is not finite or countable. That is, if $T = \{0, \infty\}$ or $T = [0, \infty\}$ for some value k. A state space S is the set of states that a stochastic process can be in. The states can be finite or countable hence the state space S is discrete, that is $S = 0,1,2,..,n$. Otherwise, the space S is continuous.

1.2.2. Markov Chains

Markov chains are a type of stochastic process with special property that probabilities involving how the process will evolve in the future depend only on the present state of the process, and so are independent of events in the past. A stochastic process is said to have the Markovian property if;

$$P\{x_{t+1} = j/x_0 = k_0, x_1 = k_1, \ldots, x_{t+1} = k_{t+1}, x_t = i\}$$

In other words, this Markovian property says that the conditional probability of any future "event", given any past "event" and the present state $x_t = i$, is dependent of the past "event" and depends only upon the present state. The conditional probabilities $P\{x_{x+t} = j/x_{t=i}\} = P_{ij}$ is the transition probability matrix.

1.2.3. Transition Probability Matrix

The conditional probabilities $P\{x_{x+t} = j/x_{t=i}\} = P_{ij}$ are called transition probabilities and can be arranged in the form of a $n \times n$ matrix known as the transition probability matrix such that it is given by;

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & p_{22} & p_{23} & & p_{2n} \\ & \vdots & & \ddots & \vdots \\ p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn} \end{pmatrix}$$

The transition matrix has the following properties,

1) $P_{ij} > 0$ for all i and j.
2) For all i and j, (i.e.) sum of the element in each row is equal to 1. This is true because the sum represents total probability of transition from state i to itself or any other state.
3) The diagonal element represents transition from one state to same state.

1.3. Properties of Markov Chains and Classification of States

1.3.1. Reducibility

A state j is said to be accessible from a state i (written $i \rightarrow j$) if a system started in state i has a non-zero probability of transitioning into state j at some point. Formally, state j is accessible from state i if there exists an integer $n_{ij} \geq 0$ such that;

$$Pr\{x_n = j\,/x_0 = i\} = P_{ij}^{n_{ij}}x > 0$$

This integer is allowed to be different for each pair of states, hence the subscripts in n_{ij}. Allowing n to be zero means that every state is defined to be accessible from itself. A state i said to communicate with state j (written i and $i \leftrightarrow j$). A set of states' C is a communicating class pair of states in C communicates with each other, and no state in C communicates with any state not in C. It can be shown that communication in this sense is an equivalence relation and thus that communicating classes are the equivalence classes of this relation. A communicating class is closed of leaving the class is zero, namely that if i is in then j is not accessible from i. A Markov chain is said to be irreducible if its state space is a single communicating class; in other words, if it is possible to get to any state from any state.

1.3.2. Periodicity

A state i has period k if any return to state i must occur in the multiples of k time steps. A Markov chain is a periodic if every state is a periodic. An irreducible Markov chain only needs one a periodic state to imply all states are a periodic.

1.3.3. Recurrence

A state i is said to be transient if, given that we start in state i, there is a non-zero probability that we will never return to i. Formally, let the random variable T_i to be the first return time to the state i (the "hitting time");

$$\mathrm{T_i} = \inf\{\mathrm{n} > 1: \mathrm{x_n} = \mathrm{i}/\mathrm{x_0} = \mathrm{i}\}$$

so that the number $f_{ii}^n = \Pr(T_i = n)$ is the probability that we return to state i after the n steps. Therefore, state i is transient if:

$$Pr(T_i < \infty) = \sum_{n=1}^{\infty} f_{ii}^{n} < 1$$

State i is recurrent (or persistent) if it is not transient.

Recurrent states are guaranteed to have a finite hitting time.

1.3.4. Ergodicity

A state i is said to be ergodic if it is a periodic and positive recurrent. In other words, a state i is ergodic if it is recurrent, has a period of l and it has finite mean recurrence time. If all states in an irreducible Markov chain are ergodic, then the chain is said to be ergodic. It can be shown that a finite state irreducible Markov chain is ergodic if it has a periodic state. A model has the ergodic property if there's a finite number N such that any state can be reached from any other state in exactly N steps. In case of a fully connected transition matrix where all transitions have a non- zero probability, this condition is fulfilled with $N = 1$. That is, a Markov chain is ergodic if there exist some finite k steps such that; $P\{x_{t+k} = j/x_t = i\} = 0$ for all i *and* j.

1.3.5. Absorbing States

A state i is called absorbing if it is impossible to leave this state. Therefore, the state i is absorbing if and only if; $P_{ii} = 1$ *and* $P_{ij} = 0$ *for* $i \neq j$.

If every state can reach an absorbing state, then the Markov chain is an absorbing Markov chain. In an absorbing Markov chain, a state that is not absorbing is called transient.

1.3.6. Homogeneity and Steady State

A Markov chain is homogeneous if its transition probabilities do not change over time. That is the probability of going from i to state j at time $t = 1$ is equal to the probability of going from state i to state j in some future period. Let the limiting distribution be v. We require the Markov kernel to be primitive. A Markov kernel is primitive if there exist an n such that $P_{ij} > 0$, for all values of i *and* j. If P has a primitive Markov kernel on finite space with limiting distribution V, then uniformly for all distribution of V is given by;

$$\lim_{n\to\infty} vp_{ij}^{n} = V$$

V is also known as stationary distribution or a steady state.

1.4. Measure for Existence of Steady State

The Chapman-Kolmogorov equations enables one to compute the P_{ij}^{n+m} transition probabilities. These equations should be interpreted as computing the probability of starting in state i and ending in state j in exactly $n + m$ transitions through a path which takes into state k at the nth transition. The equation is:

$$P_{ij}^{n+m} = \sum P_{ij}^{n} P_{kj}^{m} \; for \; n, m \geq 0 \; \forall \; i, j$$

We use the Chapman-Kolmogorov equations in order to test the limiting probabilities of Markov chains. We compute the P^n transition probabilities for $i = j$ and $k = 0$. We assume that the time periods are indifferent.

1.5. Canonical Form of the Transition Matrix

Based on the classifications of the states, the probability transition matrix P can be partitioned into its canonical form. Let an absorbing Markov chain with transition matrix P have transient states and r absorbing states. Then

$$P = \begin{pmatrix} Q & R \\ 0 & I \end{pmatrix}$$

Where;

$Q_{t\times t}$ Matrix,

$R_{t\times r}$ Matrix,

$0_{r\times t}$ Zero Matrix, and $I_{r\times r}$ Identity matrix.

Thus, Q describes the probability of transitioning from some transient state to another while R describes the probability of transitioning from some transient state to some absorbing state.

1.6. Fundamental Matrix

A basic property about an absorbing Markov chain is expected number of visits to a transient state j starting from a transient state i (before being absorbed). The probability of transitioning from i to j in exactly K steps is the (i, j) entry of the Q^K. Summing this for all K(from 0 to ∞) yields the desire matrix, called the fundamental matrix and denoted by N. It is easy to prove that

$$N = \sum_{K=0}^{\infty} Q^K = (I_t - Q)^{-1}$$

Where, I_t is the $t \times t$ identity matrix. The i, j entry of a matrix N is the expected number of times the chain is in state j, given that the chain started in state i.

Markov Chain models are useful in studying the evolution of systems over repeated trials. The repeated trials are often successive time periods where the state of the system in any particular period cannot be determined with certainty. Rather, transition probabilities are used to describe the manner in which the system makes transitions from one state to the next like from low, medium to high. It helps us to determine the probability of the system (patient flow) being in a particular state at a given period of time.

1.7. Literature Review

Markov chain models has been employed in real world problems health care problems included. An extensive literature has been done on Markov chain models to describe the stochastic dynamics nature of patient flow in health care; First and foremost, Markov chain models has been applied in continuous time period to analyze length of stay for older patients who were transported between their home and nurses' location [9]. Different version of Markov chain models was developed with discrete time period for ward admittance and capacity development in care system [10]. In their study, treatment was provided both in society (home care) and health centers. Markov chain models has been used in modelling patient transfer in an intensive care unit in hospitals in other countries like Columbia [11-13]. Besides, Markov chain models has also been used in order to give the length of stay at the cardiac surgery department of Dutch Hospital [14], in their study they considered different steps in the process and provide an approximation of likelihood that the event occurs in some order. Markov chain modeling has been implemented for patient flow in which three of the cares were considered as the states, i.e., severe, long stay, rehabilitative care [15]. A Discrete Time Markov chain (DTMC) has been applied to assess the re-admission probabilities of patients [16]. Further, (DTMC) was applied to predict the number of inpatients, demonstrating how their model attained superior predictability compared to other methods such as seasonal autoregressive integrated moving average (SARIMA) model [17]. Patient flow can be seen as Markov chain because it involves movement of patients of patients from one department to another. Markov chain models has been used to assess the care process of doctor's consultation [13]. Their study included 5 states *waiting, nurse care, examination, imaging, checkouts*. In their study historical data were used to derive the transition probability among the states. Patient flow structure can be modeled as Markov chain process as it specifies the states and possible transitions between them [18].

Basically, from the above reviewed literature materials, their studies have not provided an analysis of wider scope of a hospital care process. In this paper, we use Markov chains to analyze patient flow through the entire hospital system.

2. Materials and Methods

This paper uses a case study design. The case study was Kapsabet County Referral Hospital. The hospital is a level 5 government health facility located in Kapsabet town, Nandi County along the Eldoret - Kisumu Highway. The hospital was formerly known as the Kapsabet District Hospital. The facility manages the illness cases filtered from sub county and dispensaries in the entire county of Nandi. The data that was used in this study, was the patient flow population received at Kapsabet County Referral Hospital from January 2018 to December 2019. The data of the patients included were the general medical patients. This is because this proportion make up the largest proportion of the patients received in this hospital an approximate of 89 percent of the total patients. Data for all the months were included except for March and April 2018, as this is when the medical practitioners at this hospital were on strike and there were no patients observed during this period. R software was used in analysis.

2.1. Model Development

We suggested to apply Markov chain model to show the patient flow behavior as our objective by constructing transition probability matrix from the patient population data system based on the days of the week. The transition matrices had the five states i.e., High, Medium, Low and Very Low depending on the population of the patients received weekly. The transition probability matrices were then used obtain steady state for the transition matrices. The steady states were the observed that it displayed a pattern of the patient flow.

2.2. Establishment of the Four State Probability Transition Matrix

The patient flow pattern was either high, medium, low or very low depending on the numbers of the patients received in the hospital on the days of the week. At times, we could have a flow being so high, then it transitions to either medium, low or very low. We used the flow pattern as the Markov states, we had this pattern put to a probability matrix using the following limit categories. The population was classified as either high, medium, low or very low using these categories:

Very low (V.L) $\left(k \geq \mu - \frac{\delta}{2}\right)$ $[0 - 152]$

Low (L) $\left(\mu - \frac{\delta}{2} \leq k \leq \mu + \frac{3}{4}\delta\right)$ $[152 - 236]$

Medium (M) $\left(\mu - \frac{3}{4}\delta \geq k \leq \mu + \frac{3}{4}\delta\right)$ $[237 - 286]$

High (H) $\left(k > \mu + \frac{3}{2}\delta\right)$ $[286 \geq \infty]$

The probability transition matrix is structured in this manner;

$$\begin{array}{c} \\ H \\ M \\ L \\ V.L \end{array} \begin{array}{c} \begin{array}{cccc} H & M & L & V.L \end{array} \\ \begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & p_{22} & p_{23} & & p_{2n} \\ & \vdots & & \ddots & \vdots \\ p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn} \end{pmatrix} \end{array}$$

The transition matrix for this study involved four states. It showed how the patient flow transitioned on weekly basis, i.e., from high, medium low and very low and from any state to another.

2.3. The n-Step Transition

The absolute probabilities at any stage where n is greater that one, is determined by use of n step transition probabilities. This is the higher order transition probabilities P_{ij}^{n}. The n step matrix shows the behavior of the patient flow population in the n step later. The repeated transitions are used to evaluate whether the transition probabilities converge over time on repeated iterations i.e., $\lim_{n\to\infty} P_{ij}^{n}$.

This is the steady state probabilities which show the probability of the patient flow increasing decreasing or remaining constant over time.

3. Results and Discussions

Different probability transition matrices were used to represent the transitions of patient flow on different days of the week. The probability matrices showed how the patient flow behave throughout the week on normal circumstances without an outburst of patient flow into the facility. The days of the week ware classified according to the received numbers of the patients i.e., either high, low, medium or very low. The numbers of the patient flow population were recorded per day, divided by the total number of patients in a week to have a probability of the flow being high low or medium. The same was done for all the states; medium, low and very low to have the following transition matrices.

3.1. Computed Probability Transition Matrices and Their Steady States

Monday to Monday next week transition probability matrix

$$P = \begin{pmatrix} 0.6120 & 0.3477 & 0.0327 & 0.0076 \\ 0.5326 & 0.3974 & 0.0549 & 0.0150 \\ 0.4210 & 0.4589 & 0.0920 & 0.0280 \\ 0.3908 & 0.4745 & 0.1021 & 0.0319 \end{pmatrix}$$

This probability transition matrix is a regular one and has a higher probability 0.612 for the state high, this probability estimates patient population flow observed on this day as being high. We then computed the future n step of this transition probability matrix. In so doing, the probability transition matrix in future n days, is given by the following matrix;

Steady state matrix

The steady state is reached at future $P^7 = P^8 = P^{10} = P^n$ steps.

$$P^n = \begin{pmatrix} 0.571 & 0.3723 & 0.04437 & 0.01153 \\ 0.5709 & 0.3723 & 0.04437 & 0.01153 \\ 0.5709 & 0.3723 & 0.04437 & 0.01153 \\ 0.5705 & 0.3721 & 0.0440 & 0.01153 \end{pmatrix}$$

Tuesday to Tuesday next week transition probability matrix

$$P = \begin{pmatrix} 0.4075 & 0.4660 & 0.1096 & 0.0169 \\ 0.3652 & 0.4788 & 0.1313 & 0.0248 \\ 0.2805 & 0.4977 & 0.1787 & 0.04310 \\ 0.1903 & 0.5026 & 0.2385 & 0.0685 \end{pmatrix}$$

The probability transition matrix has a probability of 0.4 for the high state, this probability gives us patient population flow observed on this day as being lower than those received on Mondays. We computed the future n step of this transition probability matrix. Its steady state matrix is reached at future $P^5 = P^6 = P^n$ steps, for this probability transition matrix.

Steady state matrix;

$$P^n = \begin{pmatrix} 0.3651 & 0.4774 & 0.1324 & 0.0254 \\ 0.3651 & 0.4774 & 0.1342 & 0.0254 \\ 0.3651 & 0.4774 & 0.1324 & 0.0254 \\ 0.3651 & 0.4774 & 0.1324 & 0.0254 \end{pmatrix}$$

It was observed that the flow in future n step will be with probabilities for high is 0.36, medium is 0.47, low is 0.13 and very low at 0.02 for this perpendicular day. It is examined that from steady states of probability transition matrix, Tuesday's flow probabilities are lower than on Monday's where we had highest steady state probability obtained at 0.57. The medium probabilities are rising on this day form 0.34 on Monday to 0.47 on Tuesday's.

Wednesday to Wednesday next week probability transition matrix

$$P = \begin{pmatrix} 0.4100 & 0.4626 & 0.1034 & 0.0241 \\ 0.3651 & 0.4655 & 0.1293 & 0.0401 \\ 0.2992 & 0.4597 & 0.1683 & 0.0728 \\ 0.2678 & 0.4557 & 0.1870 & 0.0895 \end{pmatrix}$$

This probability transition matrix has a high probability of 0.41 for the state high, this probability indicate that the patient population flow observed on this day as being lower than those received on Tuesday. The future n step of this transition probability matrix was computed. Its steady state transition matrix is reached at future $P^5 = P^7 = P^n$ step, it is given here;

Steady state matrix

$$P^n = \begin{pmatrix} 0.3695 & 0.4635 & 0.1270 & 0.0403 \\ 0.3695 & 0.4635 & 0.1270 & 0.0403 \\ 0.3695 & 0.4635 & 0.1270 & 0.0403 \\ 0.3695 & 0.4634 & 0.1270 & 0.0403 \end{pmatrix}$$

It is evident that in future n step, probability of the flow is high with probability 0.3695, medium is 0.483, low is 0.127 and very low with 0.04 for this day. We see that the patient flow tends toward medium according to the limits we used.

Thursday to Thursday transition probability matrix

$$P = \begin{pmatrix} 0.3531 & 0.5223 & 0.1017 & 0.0229 \\ 0.3089 & 0.5319 & 0.5319 & 0.0331 \\ 0.2420 & 0.5389 & 0.5389 & 0.0813 \\ 0.1520 & 0.5353 & 0.5353 & 0.0806 \end{pmatrix}$$

This probability transition matrix has a probability of 0.35 for the sate high, this probability gives us patient population flow observed on this day as being slightly higher than those received on Wednesdays. The future n step of this transition probability matrix was computed. Its steady state transition matrix is reached at step $P^4 = P^6 = P^n$ as shown here below;

Steady state matrix

$$P^n = \begin{pmatrix} 0.308 & 0.529 & 0.127 & 0.033 \\ 0.308 & 0.529 & 0.127 & 0.033 \\ 0.308 & 0.529 & 0.127 & 0.033 \\ 0.308 & 0.530 & 0.127 & 0.033 \end{pmatrix}$$

It is observed that in future n step the probability of flow being on this day is high with probability 0.30, medium at 0.529, low at 0.12 and very low at 0.033.

Friday to Friday probability transition matrix

$$P = \begin{pmatrix} 0.2625 & 0.5557 & 0.1434 & 0.0375 \\ 0.2284 & 0.5418 & 0.1784 & 0.0513 \\ 0.1837 & 0.5108 & 0.2317 & 0.0738 \\ 0.1459 & 0.4709 & 0.2858 & 0.0974 \end{pmatrix}$$

This probability transition matrix has a probability of 0.26 for the state high, this probability gives us patient population flow observed on this day as being lower than those received on Thursday. The future n step of this transition probability matrix was computed. Its steady state is reached when the transition matrix is at future steps $P^8 = P^9 = P^n$ as shown below;

Steady state matrix

$$P^n = \begin{pmatrix} 0.222 & 0.534 & 0.186 & 0.056 \\ 0.222 & 0.534 & 0.186 & 0.056 \\ 0.222 & 0.534 & 0.186 & 0.056 \\ 0.222 & 0.534 & 0.186 & 0.056 \end{pmatrix}$$

It is evident that the future n state probability of patient's population flow on Fridays being high is with probability 0.22, medium with 0.53, low at 0.186 and very low at 0.05.

Saturday to Saturday probability transition matrix

$$P = \begin{pmatrix} 0.2101 & 0.5405 & 0.1794 & 0.0700 \\ 0.1837 & 0.5182 & 0.2012 & 0.0996 \\ 0.1539 & 0.4804 & 0.2281 & 0.1377 \\ 0.1282 & 0.4427 & 0.2521 & 0.1770 \end{pmatrix}$$

This probability transition matrix has a probability of 0.21 for state high, this probability gives us patient population flow observed on this day is slightly lower than those on Friday. The future n step of this transition probability matrix was computed. Its steady state matrix is reached when transition matrix is at $P^{12} = P^{13} = P^n$ future steps.

Steady state matrix

$$P^n = \begin{pmatrix} 0.179 & 0.515 & 0.212 & 0.1129 \\ 0.179 & 0.515 & 0.212 & 0.1132 \\ 0.179 & 0.515 & 0.212 & 0.1129 \\ 0.179 & 0.515 & 0.212 & 0.1129 \end{pmatrix}$$

It is observed that the future n step probability of population patient flow on Saturday is high with 0.17, medium at 0.51, low at 0.21 and at very low with 0.11.

Sunday to Sunday probability transition matrix

$$P = \begin{pmatrix} 0.1558 & 0.5454 & 0.2041 & 0.0949 \\ 0.1365 & 0.5176 & 0.2191 & 0.1268 \\ 0.1194 & 0.4866 & 0.2288 & 0.1652 \\ 0.0934 & 0.4349 & 0.2405 & 0.2313 \end{pmatrix}$$

This probability transition matrix has a probability of 0.15 for the high state, this probability gives us patient population flow observed on this day as being lower than those received on Saturday. Its steady state is reached when its transition matrix is at future $P^6 = P^7 = P^n$ The future n step of this transition probability matrix was computed. This is given by;

Steady state matrix

$$P^n = \begin{pmatrix} 0.1289 & 0.503 & 0.222 & 0.1466 \\ 0.1289 & 0.503 & 0.222 & 0.1466 \\ 0.1289 & 0.503 & 0.222 & 0.1466 \\ 0.1289 & 0.503 & 0.222 & 0.1466 \end{pmatrix}$$

It is observed that in future n steps the probability of population patient flow on Sunday will be high with 0.12, medium with 0.50, low with 0.22, and very low with 0.14.

3.2. Steady State Probabilities Plot

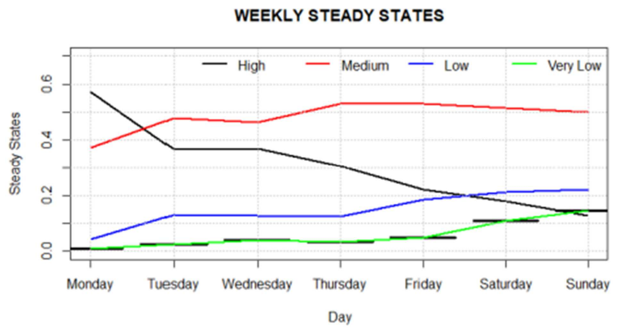

Figure 1. *The above plot displays the weekly patterns of the patient flow populations using the steady states probabilities. The curves used are; black for high, red for medium, blue for low and green for very low staedy states.*

The above plot compares the weekly patterns of the patient flow populations using the steady states probabilities of probability matrices created from weekly data. It is evident that at Kapsabet County Refferal Hospital, the patient flow is higher on Mondays (the black curve indicates the flow is high on mondays). The flow is medium from Tuesdays to Thursdays as indicated by the red curve, increasing from Monday to thursday. (From our curve, the black and the red curves slopes slightly as the blue and green rises). The rise in blue and green curves, shows that the patient flow's population decreases from Fridays to low and even into the weekends for the entire study period. This is the case even in future n steps times (weeks).

4. Conclusion

Congestion and overcrowding of patients in hospitals reduces the quality of care. Hospital administrators and their staff are always interested in alleviating congestion and overcrowding in hospitals. In this paper, we developed a patient flow assessment through an analysis of patient flow data. Transition matrices were constructed for each day in a week. This portrayed the weekly expected population of the patients received this hospital facility. Steady state transition matrices were also computed for each day of the week. This examined the flow for each day in a week. It was observed though the steady states, the patient flow had some pattern observed. The probability of patient flow in this facility tends to be higher on Mondays, medium on Tuesdays through up to Thursdays, on Fridays is when the steady states tend to decrease up to on Sundays. Through this study, some pattern can be identified, i.e., how the patient flow behaves in our hospitals. Once the pattern is identified, administrators can easily adjust and do the right planning of the staff personnel and resources in order mitigate congestion problem in our hospitals. This study validates that the Markov chains analysis is a good model to analyze the systems that change over time like patient flow. In conclusion, innovative ways are needed to explore overcrowding, the variables affecting patient flow and interventions necessary for future flow improvement.

References

[1] Houghton (2007). The American Heritage Medical Dictionary. USA: Houghton Mufflin Harcourt.

[2] Fits Gerald, G. S Toloo, J. Rego, J Ting, P. Aitken, V. Pipett (2012). 'Demand for public hospital emergency department services in Australia; 2000-2001 to 2009-2010' Emergency Medicine Australisia 24 (1); 72-78.

[3] Chiara Canta and Marie-Louise Leroux (2013). Public and private hospitals, congestion and redistribution.

[4] Hall. R., Belson, D., Murali, P., & Dessouky, M. (2006). Modeling Patient flow through the healthcare system. In R. Hall (Ed), patient flow: Reducing delay in healthcare delivery, Vol. 91: ,1-44: springer US.

[5] L. He, Y. Li and S. H Chung (2017). Markov chain based modeling and analysis of colonoscopy screening process. 740-745.

[6] Olwanda, Easter & Shen, Jennifer & Kahn, James & Bryant-Comstock, Katelyn & Huchko, Megan. (2018). Comparison of patient flow and provider efficiency of two delivery strategies for HPV-based cervical cancer screening in Western Kenya: a time and motion study. Global Health Action. 11.1451455. 10.1080/16549716.2018.1451455.

[7] Aeenparast, Afsoon & Farzadi, Faranak & Maftoon, Farzaneh & Yahyazadeh, Hossein. (2019). Patient Flow Analysis in General Hospitals: How Clinical Disciplines Affect Outpatient Wait Times. Hospital Practices and Research. 4.128-133.10.15171/hpr.2019.26.

[8] Tavakoli, M., Tavakkoli Moghaddam, R., Mesbahi, R. (2022). Simulation of the COVID-19 Patient flow and investigation of future patient arrival using a time series prediction model: A real case study. Med. Biol. Eng Comput 60, 969-990 (2022). https://doi.org/10.1007/s11517-022-02525-z

[9] Haifeng, Xie, Chaussalet, T. J., and Millard, P. (2006). A model-based approach to the analysis of patterns of length of stay in institutional long-term Care. Information technology in biomedicine *IEEE* transactions on 10 (3); 512-518.37.

[10] Garge, L., Mc Clean, S., Meenan, B., Millard P., (2010). A non- homogeneous discrete time Markov model for admission scheduling and resource planning in cost and capacity constrained health care system. Health care management science 13 (2): 155-169.

[11] Cochran, J. K., and Roche, K. T (2009). A multi class queuing network analysis methodology for improving hospital emergency department performance. Computer and operation research, 36 (5): 1497-1512.

[12] Cote, M. J., & Stein, W. E (2007). A stochastic model for a visit to doctor's office. Mathematical and computer modeling, 45 (3-4): 309-323.

[13] Perez A., Chan, & Dennis W. (2004). Predicting the length of stay of patients admitted for intensive care using first step analysis. Health care management science, 127-138.

[14] Akkerman, R., Knip, M. (2004). Reallocation of beds to reduce waiting time for Cardiac surgery. Health Care Management Science, 119-126.

[15] Mc Clean, S. I., MC Alea, B., & Millard, P. H (1998). Using Markov reward model to estimate spend down cost for geriatric department. Joper Res Soc 49 (10): 1021-1025.

[16] Nicola Bartolomeo, Paulo T., Annamaria M., and Gabriella S., (2008). A Markov model to evaluate hospital re-admission BMC Medical Research Methodology 8 (1) 23.

[17] James, R Broyles, Jeffery K Cochran and Douglas C Montgomery (2010). A statistical Markov chain approximation of transient hospital inpatient inventory. *European journal of operational research, 207 (3): 1645-1657.*

[18] Ross, S. M (2009). Stochastic process (2*ndEd.*) *New Delhi; John Wiley and sons.*