

## Abstract

In this study, I examined the practices, resources, and challenges in mathematics word problem solving (MWPS) among the Africa Swahili-speaking refugee high school students in the United States. Specifically, I investigated the language practices and linguistic resources the participants used during MWPS, as well as the linguistic challenges they faced. I also explored the mathematics practices and mathematical resources the participants used as well as the mathematical challenges they faced during MWPS. Lastly, I determined the role of the language practices and/or resources (LPRs) in the participants' mathematical processes.

To accomplish this study, I used a language background survey (LBS) and task-based interviews which were administered to 12 participants who were selected through criterion purposive sampling technique. The tasks were three problems adopted and modified from the National Assessment of Education Progress (NAEP)-released algebra problems. Guided by a Vygotskian perspective of mathematics practices, I allowed the participants a safe translanguaging space as they solved the problems. I then studied how they used their language and mathematics practices, linguistic and mathematical resources, and I noted the linguistic and mathematical challenges they faced in the process.

The analysis revealed that the participants faced various mathematical and linguistic challenges, and they also drew on their LPRs to comprehend the problems, communicate their understanding, develop their mathematics practices, and as a means of identifying with some meaningful social groups. The findings of this study showed that bi-/multilinguals translanguage in mathematics where they use their LPRs in an integrated manner, not in isolation. Since bi-/multilingual students draw on various discursive practices, their mathematics practices are oftentimes informal, making it difficult to demarcate between the students' everyday and

mathematics practices (Barwell, 2013). Also, the findings showed that bi-/multilinguals need support to use their LPRs in a mathematical sense and to develop more formal mathematical practices.

The findings of this study have implications on the validity of assessments, and how teachers can be prepared to teach bi-/multilinguals, even when they don't share the students' home languages. Drawing on the work of Civil (2012) and Sigley and Wilkinson (2015), I argue that valid assessments would have to valorize bi-/multilingual students' ways of communicating mathematically, even those that may not seem precisely mathematical. Moreover, teachers are to be cognizant of the bi-/multilingual students' ways of mathematical communication and determine ways they could use those ways to enhance the students' learning of mathematics. I also present de Jong et al.'s (2013) conceptual framework that can be used to enhance the preparation of mainstream teachers to support ELLs in content areas. This study suggests the need for further research on translanguaging in mathematics classrooms and how teachers can implement pedagogies that support translanguaging to enhance learning. There is also a recommendation for studies investigating the kinds of professional development mainstream mathematics teachers would need to be effective in the instruction and assessment of students whom they don't share the home language. Also, there is need for further research on how students solve problems and generalize and how they can be supported to develop these processes.

An Investigation of Practices, Resources, and Challenges in Mathematical Word Problem Solving among Swahili-speaking African High School Bi-/Multilingual Students in the United States

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## Acknowledgement

Growing up as a young girl in Mutomo Village, Kitui County, Kenya, I remember how I struggled with mathematics. But my dad kept encouraging me that I could make it. My dad's words of encouragement were a prophesy that sounded like one of the many names he used to call me. That prophesy has now come to pass; I love mathematics and I am a proud mathematics educator. I want to thank God in whom we live, move, and have our being. I thank God for Professor Joanna O. Masingila. I am forever grateful to you, Jo, for being the best mentor I could have asked for. When I came to Syracuse I was naïve and timid but because of your unfailing mentorship, I am now ready to mentor others and make this world a better place.

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## **List of Abbreviations and Acronyms**

ACF	Adventist Christian Fellowship
AMTE	Association of Mathematics Teacher Educators
DIF	Differential Item Functioning
ECR	Extended Constructed Response
ELLs	English Language Learners
ESL	English as A Second Language
ICE	International Conference on Education
KCPE	Kenya Certificate of Secondary Education
LBS	Language Background Survey
LPRs	Language Practices and/or Resources
MWPS	Mathematics Word Problem Solving
MWPs	Mathematics Word Problems
NAEP	National Assessment of Educational Progress
NCES	National Center for Education Statistics
NCTM	National Council of Teachers of Mathematics
PME-NA	North American Chapter of the International Group for the Psychology of Mathematics Education
PrA	Problem A
PrB	Problem B
PrM	Math Eliciting Task
TIMSS	Third International Mathematics and Science Study

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## Chapter 1: Introduction

### **Background of the Study**

Current reforms in mathematics education view communicating mathematically as the central indicator of mathematics learning (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Students are to be able to communicate mathematical ideas and relationships in either oral or written form in the English language. The mathematical ideas and relationships that students are to develop and communicate are the mathematics practices that have been highlighted in the standards for mathematics practices. For instance, students are to be able to abstract, generalize, conjecture, test conjectures, construct arguments, and subject claims and arguments to discussion and evaluation by a classroom community (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

While communicating mathematically is central to the learning of mathematics, there has existed tension as to how mathematics practices should be defined or described (Moschkovich, 2013). Mathematics practices have been dichotomized as either everyday/academic, formal/informal, or in-school/out-of-school (Moschkovich, 2013). Traditional mathematics instruction views the relationship between school-based practices and out-of-school practices as “unidirectional”, where the out-of-school practices are to be informed by the school practices and not the other way around (Dominguez, 2011). Such dichotomies are not sufficient ways to describe mathematics practices since mathematics practices occur in multiple contexts that may be academic, workplace, playground, street selling, and home among others (Moschkovich, 2013).



In order to address the question of how mathematics practices should be described, there first a need to understand how mathematics practices are appropriated and manifested by all students, especially those for whom English is not their first language. According to Grosjean (1999) and Halai (2007), there are differences in the factors and processes involved in developing communicative competence of English language learners (ELLs) and that of their native counterparts. These findings imply that ELLs may communicate their mathematics practices in ways that are different from those of a native English speaker. Moschkovich (2013) emphasized the need for studies that seek to understand the nature of mathematics practices students use in different contexts and settings; studies that “make visible the ways that learners reason mathematically across settings” (Moschkovich, 2013, p. 264).

Moschkovich (2013) used a Vygotskian perspective and Scribner’s (1984) definition of practice as “culturally organized in nature and involving different technologies or symbols systems” (p. 265), to define mathematics practices as social, cultural, cognitive, and semiotic. Mathematics practices are socio-cultural because they originate from social interaction, where the learner is actively involved in a joint activity that “supports their appropriation of goals, focus of attention, and shared meanings” (p. 271). Mathematics practices are cognitive because they involve thinking, and they are semiotic because they involve semiotic systems such as signs, tools, and their meanings. According to a Vygotskian perspective, mathematics practices are embedded in mathematical discourse practices and meaning of utterances. These discourse practices not only involve the use of language, but also the use of other symbolic expressions, objects, and communities. Discourse practices thus entail aspects of both academic and everyday practices. In brief, mathematics practices are expressed in the meanings generated from both

everyday and academic discourse practices that students engage in while working on a joint mathematical activity.

From a Vygotskian perspective, everyday discourse practices are not obstacles to participation in academic mathematics practices, but resources for students' participation in formal mathematics practices (Moschkovich, 2013). It is therefore pertinent to study how students, especially those for whom English is not their first language, use language, symbolic expressions, objects, and other tools, as well as the role of the use of these resources in their mathematical processes. Such knowledge can inform how mathematics practices should be described, as well as how ELLs' mathematical communication can be supported.

### **Statement of the Problem**

Research has shown that students whose first language is not English face certain linguistic and cultural biases in standardized examinations such as the National Assessment of Education Progress (NAEP) (Abedi & Lord, 2001; Kieffer, Lesaux, Rivera, & Francis, 2009; Martiniello, 2009). These students, commonly known as ELLs, not only underachieve in NAEP mathematics tests, but they are also underrepresented in the nation's students' report card (Pellegrino et al., 1999). Research on mathematics and language among the ELLs has limited ELLs' identity to the acquisition of English language. The result has been the assertion that the complexity in the natural language of a mathematics text or test, in this case, English, is the cause for ELLs' underperformance in mathematics (Abedi, 2004; Barbu & Beal, 2010; Cuevas, 1984; Martiniello, 2008, 2009; Schleppegrell 2007).

Current research has problematized this model of viewing ELLs, terming it as a deficit model (Garcia, 2009; Moschkovich, 2007). Researchers are now being urged to shift from viewing ELLs in terms of their deficiency in English, into seeing them as learners possessing

certain linguistic and cultural resources, which they tap into to make sense of mathematics (Moll, 2010; Moschkovich, 1999; Yosso, 2005). There is a need for studies on what bi-/multilinguals actually do with their language, what language does to them, and what language means to them (e.g., Martinez-Roldan, 2014). Instead of comparing mathematical performance between bi-/multilinguals and monolinguals, researchers have been called to investigate in detail the students' communicative practices and report on differences that favor bi-/multilinguals, and those that are relevant to their mathematical instruction (Moschkovich, 2007). Researchers should seek to know the nature of bi-/multilinguals' language practices and the kinds of resources they draw upon to make sense of mathematics, in order to "make visible" the mathematics practices that result from their use of these language practices and linguistic resources in different contexts (Moschkovich, 2013, p. 264).

A number of studies have looked at how ELLs make sense of mathematics by drawing on various resources such as code-switching (Moschkovich, 2002, 2007), translation (Halai, 2007), gestures (Dominguez, 2005), and everyday/cultural experiences (Dominguez, 2011; Vomvoridi-Ivanovic, 2011), as well as inscriptions (Moschkovich, 2008), and various semiotic features and structures (Solano-Flores, Barnett-Clarke, & Kachchaf, 2013; Solano-Flores & Nelson-Barber, 2001; Solano-Flores, Wang, & Shade, 2016). However, ELLs do not use these practices and resources in isolation, but they employ multiple, complex, and integrated discursive practices to make sense of mathematics. The use of multiple interrelated discursive practices that cannot be easily assigned to one or another traditional definition of language, but that make up the speaker's complete language repertoire, has been termed as translanguaging (Garcia & Wei, 2014, p. 22). According to Garcia and Wei (2014), "translanguaging better captures the sociolinguistic realities of everyday life" (p. 29). Sociolinguists stress the social

nature of language and its use in varying contexts under the assumption that language is not only cognitive, but also cultural, social, and situated (Moschkovich, 2007). Most of these studies have been done among the English-Spanish speaking bilinguals at lower grade levels. English-Spanish bilinguals are only a portion of the ELL population in the United States. There is thus a need for studies addressing practices, resources, and challenges in mathematics learning among other ELL populations at higher grade-levels such as high school.

In this study, I considered ELLs' language practices as translanguaging practices and viewed mathematics practices from a Vygotskian perspective. I studied these phenomena among the Swahili-speaking African refugee high school bi-/multilingual students. According to the United States Department of State Bureau of Population, Refugees and Migration records, as of March 2017 refugees from Africa form the second largest population of refugees in the U.S., with 37.46% (14,277) out of the total refugee population of 38,111 (U.S. Department of State, 2017). The leading refugee population originates from the Near East/South Asia, with 43.56% (16,600). Moreover, many of the refugees from Africa speak Swahili as their native language since Swahili is the lingua franca for many African countries. According to these records, Swahili is the sixth of the top ten native languages spoken by the refugee communities currently living in the U.S. To these students, English is either a second or a third language, making them bi-/multilinguals. No research in mathematics education has yet investigated the practices, resources, and challenges in mathematics word problem solving (MWPS) in the context of students from the Swahili-speaking African refugee community.

By providing these students with an opportunity to translanguage, I identified the language practices and other resources the students drew on to solve three algebra word problems adopted and modified from the NAEP-1990, 1992, and 2009 released test items, as

well as the challenges they faced during problem solving. I also investigated the mathematics practices emanating from the meanings of the mathematical discourse practices the students used, together with the mathematical resources and knowledge they drew upon and the mathematical challenges they seemed to face during problem solving. Hopewell (2011) noted that there is a dearth of studies examining when, why, and how students use their linguistic repertoire. Therefore, in this study, I not only investigated the language practices and other resources (LPRs) the participants used in MWPS, but I also sought to know the role of LPRs in the participants' mathematical processes of problem solving, reasoning and proof, connecting, communication, and representation (National Council of Teachers of Mathematics [NCTM], 2000).

### **Purpose of the Study**

The purpose of this study was to investigate the practices, resources, and challenges in MWPS among bi-/multilingual high school students from the African Swahili-speaking refugee community, who recently immigrated to the U.S. Specifically, I investigated the linguistic and mathematical practices and resources the participants used during mathematics word problem solving (MWPS), as well the linguistic and mathematical challenges they faced during this process. I also investigated the role of the LPRs in the participants' mathematical processes of problem solving, reasoning and proof, communication, connection, and representation.

### **Research Questions**

This study was guided by the following research questions:

1. What language practices do the participants use during MWPS? What linguistic challenges do they encounter? What linguistic resources do they use?

2. What mathematics practices do the participants use during MWPS? What mathematical challenges do they encounter? What mathematical resources do they use?
3. What role, if any, do the LPRs play in the participants' participation in mathematical processes?

### **Significance of the Study**

This study strives to complement and advance existing research on mathematical communication by studying the practices, resources, and challenges in MWPS among a population that has not been studied before. The study also advances research on mathematics education among ELLs by employing frameworks that do not view ELLs as deficient, but as possessing certain practices and resources that they draw upon to make sense of mathematics. To the research world, the findings of this study on how bi-/multilinguals manifest their mathematics practices may contribute to the debate of how mathematics practices should be described. To curriculum developers, these findings provide information that might be used in developing a curriculum that prepares teachers to teach bi-/multilingual students. A mathematics teacher might also use the findings of this study to determine ways of supporting bi-/multilingual students develop mathematics practices.

### **Theoretical Frameworks**

This study is premised upon a Vygotskian perspective of mathematics practices, which views mathematics practices as social, cultural, semiotic, and cognitive (Moschkovich, 2013). According to this framework: (a) students develop mathematics practices as they work jointly on a mathematics task, (b) mathematics practices are embedded in the discourse practices students engage in as they solve a mathematics task, (c) the discourse practices that students engage in entail both everyday and academic practices. Everyday practices involve students drawing on their language practices and linguistic resources, while academic practices involve students

drawing on various mathematical resources. In this study, I framed language practices and resources of bi-/multilingual students as translanguaging practices and I positioned the language of mathematics as a multi-/semiotic system. In the following sections, I discuss a Vygotskian perspective on mathematics practices, bi-/multilinguals' LPRs as translanguaging practices, and the language of mathematics as a multi-/semiotic system.

### **Vygotskian Perspective on Mathematics Practices**

In this study, I use the term practice or practices in the sense used by Moschkovich (2013). Following Scribner's (1984) usage of the term practices, Moschkovich presented a Vygotskian perspective of a practice or practices as "culturally organized in nature and involving different technologies or symbols systems" (p. 265). Based on a Vygotskian definition of practice or practices, Moschkovich also defined mathematics practices as social-cultural, cognitive, and semiotic. Mathematics practices are socio-cultural because they originate from social interaction where the learner is actively involved in a joint activity. Mathematics practices are cognitive because they involve thinking, and they are semiotic because they involve semiotic systems such as signs, tools, and their meanings (Moschkovich, 2013).

A Vygotskian perspective on mathematics practices has a number of implications including (a) social interaction where learning is predominantly through joint activity, (b) goals are implicit but fundamental aspects of practices, (c) discourse is central to participation in practices, (d) meanings for words are situated and constructed while participating in practices, and (e) appropriation is a central description for learning, but learners do not simply imitate practices, they sometimes transform them (Moschkovich, 2013). According to the Vygotskian perspective, mathematics practices are not as the practices we tell students or model on the board instead, they are the practices that learners develop when they are engaged in discourse during a

joint mathematical activity. Mathematics practices are embedded in mathematical discourse practices, where discourse is more than language use since it involves other symbolic expressions, objects, and communities. Within the Vygotskian framing, mathematics practices are not purely cognitive accounts of mathematics but accounts that assume the social, cultural, and discursive nature of mathematical activity (Moschkovich, 2008). Therefore, “students are likely to need time and support as they move from expressing reasoning and arguments in imperfect form towards more academic ways of talking” (Moschkovich, 2013, p. 271).

In this study, students worked in pairs to solve two algebra word problems adopted and modified from the NAEP-1990 and 1992 released test items because NAEP assessment provides data on achievement that is tailored to students’ school experiences in the U.S. and these tests contained problems that I determined would be useful for these tasks. I asked the students to talk aloud about the problems and their solution processes. I presented the problems using different semiotic features in an attempt to explore the role different semiotic resources play on MWPS among bi-/multilinguals (Solano-Flores et al., 2013).

### **Bi-/multilinguals’ Language Practices as Translanguaging**

From a Vygotskian perspective, mathematics practices are constructed when students participate in discourse while engaged in a joint mathematics activity (Moschkovich, 2013). The mathematical discourse practices that students engage in during problem solving include the use of language(s), symbolic expressions, and other objects and visual devices. In this study, I consider language as a social and cultural practice (Palmer & Martinez, 2016); a form of action that emerges within particular social and cultural contexts (Garcia, 2009). Seeing language as a practice or an action occurring within a social or cultural context means that bi-/multilinguals are not bounded in the manner of use of their language(s). Consequently, bi-/multilingualism is not