STUDY OF NON-NORMAL OPERATORS IN A COMPLEX HILBERT SPACE

By

Justus Kitheka Mile

A Thesis submitted in partial fulfillment of the requirements for the award of the degree of Doctor of Philosophy in Pure Mathematics

Department of Mathematics and Applied Statistics

Maseno University

ABSTRACT

A normal operator has been an object of much study in operator theory and possesses many nice properties and non-normal operators in the set of all bounded operators in a Hilbert space can be classified according to their satisfying some of such properties or generalizations of some of these properties.

We have classified some of the non-normal operators and investigated the relationship between these classes. In particular, we have shown that if a bounded operator is quasinormal then it is subnormal and hence hyponormal. If an operator is hyponormal, then it is paranormal and consequently its square is paranormal but not hyponormal. If a bounded operator is paranormal, then it is k-paranormal and hence is normaloid. We have also shown that the implications cannot, in general, be reversed.

We have also obtained a set of necessary and sufficient conditions for convexoidity and characterized those operators for which the real part of the spectrum equals the spectrum of the real part of the operator generally.

Using the result and the method of Lebow A., we have obtained results which indicate a connection between spectral sets, the numerical range and normal dilation of an operator.

Using the techniques employed by Paul Halmos in the course of studying reducible operators, we have investigated the class R_1 of operators and proved that this class includes normaloid, spectraloid, paranormal, hyponormal and T+k, where the operator T is isometric or has G_1-property, or hyponormal and k is compact.