MATHEMATICAL MODEL TO DETERMINE THE CONCENTRATION OF FLY-ASH AT SONY SUGAR COMPANY IN KENYA.

By

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ABSTRACT

Mathematical modeling continues to attract attention of applied mathematician, physicist, engineers, biologists, research institutions and others. This study focuses on air pollution concentration: a case study of fly-ash at Sony Sugar Company, Kenya. Major sources of pollutants include industrial effluents, agricultural chemical and traffic vehicle exhaust. Air pollutants are dispersed by wind into the atmosphere and within an enclosure. At Sony Sugar Factory, air pollution is mainly contributed by fly-ash from unburned bagasse due to insufficient combustion. This has had a major negative impact on the surrounding, for instance, on vegetation and public health. The objective of this study was to determine the concentration of fly-ash within the boiler chimney at Sony Sugar Factory. We used a numerical analysis approach that involves two-dimensional finite elements to determine the concentration of the pollutant. We assumed the cross-section of the chimney is polygonal in shape, which was discretized into small triangular elements. The result obtained may be useful to engineers, environmentalists, agricultural and Industrials sectors, and research institutions in the assessments, analysis and predictions of temperature distributions and variation of pressure that can minimize the pollutant concentration and volume.

Chapter 1

Introduction

1.1 Background of The Study

Numerical approximations and solutions are the basis for many mathematical models of physical, chemical and biological phenomena, and their use has also spread into economics, financial forecasting and other fields. Because exact analytical solutions are generally not easy to find, it is often necessary to resort to numerical methods to find approximate solutions of these partial and ordinary differential equations, in order to investigate the predictions of the mathematical models.

Our study of determination of concentration of air pollutant within the boiler chimney was triggered by the research in 2010 in Thailand [21]. In that research, numerical experiment of air pollutant concentration in the street tunnel was conducted by Montri Thongmoon[21]. Consequently, we used interpolation techniques to determine the nodal concentration of fly-ash within the boiler chimney which is the domain of our study. In [21] three-dimensional advection-diffusion equation was used to determine concentration of air pollutant in the street-tunnel. Literature of the study is in chapter two.

In chapter three, examples and derivation of the model is explained. Sousa[25] studied the conditions that ensures stability for various finite difference schemes for advection-diffusion equation, Dehghan [14] defines and calculates stability conditions for several numerical methods e.g forward in time center in space(FTCS); forward in time backward in space(FTBS); Lax-Wendroff; backward in time center in space (BTCS); and Crank-Nicolson three-dimensional advection-diffusion equation and compares the numerical and analytical solutions. As a matter of emphasis, common effluent that spring up with industrialization, agricultural, poverty and economic demand of both the developed and developing nations are traffic exhaust, dust, hydrogen sulphide, sulphur dioxide, carbon and particulate matter e.g. fly-ash.

As non-point source pollution has garnered more attention in recent years, governmental agencies, academic and research institutions, and commercial consulting firms have developed methods of assessing pollution from non-point sources. Many of these methods have involved the development of computer based models for automated, reliable , analysis.More recently, some of these models have been linked with geographic information system(GIS) for ease of data management.

Partial differential equations not only find application in applied mathematics, but also have leverage in modeling and research.Thongmoon [21] conducted outdoor numerical experiment of air pollutant in the street tunnel. In their research, three-dimensional advection-diffusion equation was applied to model concentration of air pollutant. Since this is partial differential equation, they solved the equation using finite difference method. It is important to emphasize, that fly-ash is not the only pollutant at Sony-Sugar factory.Additionally, there is sulphur dioxide, nitrogen dioxide and sugar dust.Our main reason for choosing Fly-Ash was its nature.Its a particulate pollutant this makes it visible in the company set up and the surrounding environment.Because of its nature , it settles on roofs and on crops this inhibits gaseous exchange and flowering of plants. The people in the vicinity often complain of respiratory problems and corrosion of their roofs.Ocassionally, the public experience black rains after a long dry spell.

1.1.1 Definition of terms

Pollutant:- Includes any substance whether liquid, gaseous or solid which may directly or indirectly alter the quality of any element of the receiving environment; hazardous or potentially hazardous to human health or the environment, and includes objectionable odors, radio activity, noise, temperature change or physical, chemical or biological change to any segment or element of environment.

Pollution:- Means any direct or indirect alteration of the physical, thermal, chemical or biological or radio-active properties of any part of the environment by discharging, emitting, or depositing wastes so as to affect any beneficial use adversely, to cause a condition which is hazardous or potentially hazardous to public health, safety or welfare, or to animals, birds, wildlife, fish or aquatic life or plants or to cause concentration of any condition, limitation, or restriction which is subject licence under this act[EMCA](1999)

Bagasse:- Is a solid waste obtained after extraction of juice. It is used as boiler fuel in generation of steam. Small quantities of same are sold to farmers in Kisii for mushroom production. Currently, the production of bagasse is more than is demanded. This has led to formation of a large heap of the same outside the boiler area. This state of affairs has aggravated the fly-ash problem which is a health risk(bagacillo) to the workers and the surrounding communities. In addition, it made it costly to the company to maintain their roof and also the vegetation is affected.

Boiler ash: This is a waste product from the combustion of bagasse. It is a rich source of minerals and is used in the cane fields as fertilizer.

Molasses: This is a by-product of sugar production process. Molasses serves as a valuable feedstock for alcohol production and downstream chemical complex of alcohol based industries.

1.2 Statement of the problem

Fly-ash is insufficiently burnt bagasse that is released into the air by sugar factory chimneys. SonySugar for example, releases alot of it into the atmosphere surrounding the factory. This has negatively impacted on the surrounding air, vegetation and the general public health.Additionally, black rains have often been experienced during the first rains that come soon after the dry period, an indication that there is alot of fly-ash in the air surrounding the factory.There is blurred visibility experienced mostly during the dry spell.Also, there are many people suffering from respiratory problems e.g asthma, a very common phenomena during the dry periods. These could be factors of fly-ash pollution produced by the SonySugar Factory chimney. It has been observed that the leaves of vegetation are very dirty and so are the roofs during the dry periods, a clear indication of the presence of this high fly-ash production.

Because of these problems, there is need to investigate the quantity of fly-ash produced by the SonySugar factory chimney, and possibly come up with a solution that could reduce the quantity of this pollutant.

1.3 Objectives of the study

Our main objective was to:
Investigate the effect of Temperature and Pressure on Fly-ash production at SonySugar factory.

The Specific Objectives were to:

- To investigate the effect of chimney Temperature variation on the production of fly-ash at SonySugar factory.
- To investigate effect of the chimney Pressure variation on the production of fly-ash at SonySugar factory.

1.4 Research Methodology

This study used two-dimensional finite element equation to determine the effect of Temperature and Pressure on Fly-ash production by the SonySugar factory chimney.Quantity of the pollutant was incorporated in parts per million(PPM),Temperature in degrees centigrades and pressure variation in bars(br). In computation , mathematica and MaTLaB was used to solve the matrices and graphs to analyze the concentration of the pollutant. Product moment correlation coefficient was used to determine the stability of models. The chimney cross-section area was assumed to be within which there were small triangular elements each joined by nodes and analyzed the stability of the scheme. Another assumption was that the chimney cross-section was polygonal in shape with fixed concentration conditions given at the vertices within the chimney. Our parameters from Sony-Sugar daily Boiler Log Book report.

1.5 Significance of the study

This study supports the eighth millennium development goals(MDGs) to be achieved by 2015, that required the World's main development challenges(UNDP,2015).The MDGs are drawn from the actions and targets contained in the millennium declaration that was adopted by 189 Nations and signed by 147 heads of states and Governments during the UN millennium summit in 2000.The study support a strong comprehensive legislative framework for the management of the environment in the County and Country.It therefore support the legislation provided for the creation of the National Environment Management Authority(NEMA) and also the Environmental Management and Co-ordination Act(EMCA) of 1999. The result obtained will be useful in the determination, assessment and prediction of concentration of pollutants at a given point within the chimney, in the analysis and variation of temperature and pressure distribution within the furnace. Additionally, engineers and environmentalist may find it useful in specification of equipments design, and environmental management assessment and evaluation. Moreover, the model with help learning institutions and industrial management in research and planning.

Chapter 2

Literature Review

2.1 Introduction

Air pollution come from both natural and man made sources. Though globally man made pollutants, from combustion, construction, mining, agriculture and warfare are increasingly significant in air pollution equation. Motor vehicle emissions are one of the leading causes of air pollution. China, USA,Russia, Mexico, and Japan are the world leaders in air pollution emissions. Principal stationary sources include chemical plants, coal-fired plants, petrochemical plants, nuclear waste disposal activity e.t.c. In addition, agricultural air pollution comes from contemporary practices which include felling and burning of natural vegetation as well as spraying of pesticides and herbicides.Continuous development and increase of population in the urban areas, a series of problems related to environment such as deforestation, release of toxic materials, solid waste disposals, air pollution and many more, have attracted attention much greater than ever before. The problem of air pollution in cities has become so severe that there is a need for timely information about changes in the pollution level. Air pollution dispersion is a complex problem. It covers pollutant transport and diffusion in the atmosphere. Pollutant dispersion in the atmosphere depends on pollutant features, meteorological, emission and terrain conditions. Physical and mathematical models are developed to describe the air pollution dispersion. Physical models are small scale representations of the atmospheric flow carried out in wind tunnels. Mathematical models are divided in to statistical and deterministic models. The African Refiners Association and the World Bank are working with other organizations and the governments of Sub-Saharan Africa to study the costs and benefits of reducing pollutant emissions from gasoline and diesel fuel(Robinson and Hammitt,2009)[23]. To support this effort, the World Bank contracted with ICF International to conduct two studies: one on the health benefits associated with improvements in air quality, and the second on the costs of reducing pollutants in fuels produced in Sub-Saharan Africa.

Pollution as a consequence of industrialization, economic and social demands has become a major world concern.Recently, the Copenhagen conference in Denmark and the Kyoto protocol, Japan, focussed on climate change and emissions, resulting in looming global warming(2010). In Kenya, in the year(2010) agricultural chemicals disposal massively destroyed thousands of aquatic lives, particularly fish, in lake Naivasha.The industrial revolution precipitated rapid industrialization, rapid growth of urban centers and a greater dependence on fossil fuels brought about an increase in emissions of harmful pollutants into the atmosphere. Pollution of the atmosphere affects the lives of millions of people in all parts of the world, especially those living in large industrialized cities. The World health organization lists some of the major environmental problems of urban and industrial areas and their surroundings as: unpleasant fumes and odors, reduced visibility, injury to human health and crops, damage to property by dust and corrosive gases.

In Kenya, some studies on air pollution have been going on since 1993. Most of these studies have been carried out in Nairobi which is a rapidly growing city with a population of over 2million people. It is the capital of kenya and hence it is the commercial as well as industrial center of the country. Commercial activities are concentrated mainly within the city center whereas most of industrial activities are located to the south-east. Common air pollutants include carbon monoxide, nitrogen oxide, sulphur dioxide, lead and Total Suspended Particulates(TSP) which include dust, smoke, pollen and other solid particles(i.e. fly-ash) from unburnt bagasse.Most of these substances occur naturally in low (background) concentrations, when they are largely harmless; they become pollutants only when their concentrations are relatively high compared to the background value and begin to cause adverse effects. These concentrations vary widely depending on the sources of pollution and their distribution, meteorological conditions and the topographical features in the vicinity. The amount of pollutant in the air is expressed in parts per million (PPM). Previous studies have shown that in general,where there is air pollution, TSP represent the most serious immediate threat to human health amongst air pollutants. The pollutant dispersion in the atmosphere depends on pollutant features, meteorological, emission and terrain conditions. Physical and mathematical models are developed to describe the air pollution dispersion. Physical models are small scale representations of the atmospheric flow carried out in wind tunnels. Mathematical models are divided in to statistical and deterministic models.

2.2 Historical Background of Air pollution

Air pollution has always accompanied civilization. Soot found on ceilings of prehistoric caves provides ample evidence of the high levels of pollution that was associated with inadequate ventilation of open fires[21], the forging of metals appears to be key turning point in the creation of significant air pollution levels outside the home. Core samples of glaciers in Greenland indicate increase in pollution associated with Greek, Roman, Chinese metal production. King Edward 1 of England banned the burning of sea-coal by proclamation in London in1272, after it smoke had become a problem. Air pollution would continue to be a problem in England especially later during the industrial revolution, and extending into the recent past with great Smog of 1952.

A "poor air quality" sign posted over a highway, in Salt Lake City last year,2013. (AP Rick Bowmer) GENEVA (AP) Carbon dioxide levels in the atmosphere reached a record high in 2013 as increasing levels of man-made pollution transform the planet, the U.N. weather agency said .As the heat-trapping gas is blamed for the largest share of global warming, carbon dioxide rose to global concentrations of 396 parts per million last year, the biggest year-to-year change in three decades. That's an increase of 2.9 ppm from the previous year and is 42 percent higher than before the Industrial Age, when levels were about 280 parts per million (World Meteorological Organization (WMO), 2013)[32].

Based on the current rate, the world's carbon dioxide pollution level is expected to cross the 400 ppm threshold by 2016, said WMO Secretary-General Michel Jarraud. That is way beyond the 350 ppm that some scientists and environmental groups promote as a safe level and which was last seen in 1987.

Greenhouse gas emissions are building up so fast that top climate scientists are becoming increasingly skeptical that countries across the globe will meet the goal they set at the 2009 Copenhagen climate summit of limiting global warming to about another 3.6 degrees Fahrenheit (2 degrees Celsius) above current levels. In a draft report the United Nations' Intergovernmental Panel on Climate Change said it is looking more likely that the world will shoot past that point and by mid-century temperatures will increase by about another 3.6 degrees Fahrenheit (2 degrees Celsius) compared to temperatures from 1986 to 2005. And by the end of the century that scenario will bring temperatures about 6.7 degrees warmer (3.7 degrees Celsius), it said. "We know without any doubt that our climate is changing and our weather is becoming more extreme due to human activities such as the burning of fossil fuels," Jarraud said. "Time is not on our side, for sure." To address the challenge, U.N. Secretary-General Ban Ki-moon has invited heads of state and other leaders to a climate change summit in New York on the sidelines of the annual U.N. General Assembly. President Barack Obama has said he will attend to help spur new commitments from governments, industry and civil groups for reducing greenhouse gas emissions ahead of next year's(2015) global climate talks in Paris. The WMO report said the rate of ocean acidification, which comes from added carbon absorbed by oceans, "appears unprecedented at least over the last 300 million years." Between 1990 and 2013, carbon dioxide and other gas emissions caused a 34 percent increase in the warming effect on the climate, the report said.The warming effect, or "radiative forcing," measures the net difference between the sunlight that the Earth absorbs and the energy it radiates back into space. More absorption leads to higher temperatures. After carbon dioxide, methane has the biggest effect on climate. Atmospheric concentrations of methane reached a new high of 1,824 parts per billion in 2013, up 153 percent from pre-industrial levels of about 700 parts per billion.About 40 percent of the methane comes from natural sources such as termites and wetlands, but the rest is due to cattle breeding, rice agriculture, fossil fuel burning, landfills and incineration, according to the agency.

2.3 Air Pollution Modeling

Air pollution models play an important role in science. Air pollution models are the only methods that quantify the deterministic relationship between emissions and concentrations/ disposals, including the consequences of past and future scenarios and the determination of effectiveness of abatement strategies. This makes air pollution models indispensable in regulatory, assessment, research, and forensic applications. The concentration of substance in the atmosphere is determined by; (1) transport,(2) diffusion,(3) chemical transportation,(4) ground deposition. Transport phenomena,characterized by the mean velocity of fluid, have been measured and studied for centuries. For example, the average wind has been studied by man for sailing purposes. The study of diffusion (*turbulent*) motion is more recent.Among the first articles that mention turbulence in the atmosphere, are those by Taylor (1915, 1921).

One of the first challenges in the history of air pollution modeling was understanding of the diffusion properties of plumes emitted from large industrial stacks. For this purpose, a very successful yet simple model was developed, the Gaussian plume model. This model was applied for the main purpose of calculating the maximum impact from source. The model was formulated by determining experimentally the vertical and horizontal spread of the plume, measured by standard deviation of the plume's spatial concentration distribution.

(1)A comprehensive review of air pollution aerodynamics was recently compiled by Meroney (2004), addressing the wide range of methods that exist for predicting pollutant dispersion, ranging from field tests and wind tunnel simulations to semi-empirical methods and numerical simulations with Computational Fluid Dynamics (CFD). Several field tests have been conducted in the past (Barad 1958, Wilson and Lamb 1994, Lazure et al. 2002, Stathopoulos et al. 2002, 2004). They are very valuable because they are conducted in the real atmospheric boundary layer and provide information on the real complexity of the phenomenon[11]. (2)As opposed to field tests, wind tunnel modelling allows controlled physical simulation of dispersion processes (Halitsky 1963, Huber and Snyder 1982, Li and Meroney 1983, Saathoff et al. 1995, 1998, Leitl et al. 1997, Meroney et al. 1999, Stathopoulos et al. 2002, 2004)[11].Drawbacks of wind tunnel tests are that they can be time-consuming and costly, that they are not applicable for light wind conditions,and that scaling similarity can be a difficult issue. (3)Semi-empirical models, such as the Gaussian model

(Turner 1970, Pasquill and Smith 1983) and the so-called ASHRAE models (Wilson and Lamb 1994, ASHRAE 1999, 2003) are relatively simple and easy-to-use, at the expense of limited applicability and less accurate estimates[11]. (4)The Gaussian model, in its original form, is not applicable when there are obstacles between the emission source and the receptor,and the ASHRAE models only evaluate the minimum dilution factor on the plume centreline.Numerical simulation with CFD offers some advantages compared to other methods: it is often said to be less expensive than field and wind tunnel tests and it provides results of the flow features at every point in space simultaneously[11]. (5) However, CFD requires specific care in order for the results to be reliable.CFD simulations of turbulent flow based on the Reynolds-averaged Navier- Stokes (RANS) equations or with LES should at least always be validated by comparison with high-accuracy experimental data.CFD simulations of pollutant dispersion are further complicated by the fact that the knowledge of the turbulent Schmidt (Sct) is required prior to the simulation. (6) Riddle et al. (2004) simulated dispersion of emissions from a 30 m high isolated stack under neutral open country conditions using Fluent 6.1 with Sct = 0.3. Tang et al. (2005) used Fluent 6.2 to simulate plume dispersion from a ground level source in an open field.

In 2010, Montri Thongmoon[28] conducted numerical experiment of air pollutant concentration in the street tunnel.They used in their research a three-dimensional advection-diffusion equation of air pollutant was applied with a FTCS finite difference scheme.In 2011 Konglok and Pochai used fractional step method for solving the smoke dispersion model.In our study we used a two-dimensional Finite-Element method to determine concentration of air pollutant Fly-Ash at Sony- Sugar Company. In this study the researcher developed Mathematical model for determining concentration of air pollutant at Sony- Sugar Company, Kenya.

Chapter 3

Basic Concepts

See [?] In this study we determined the nodal concentration of air pollutant within the boiler-chimney using a numerical technique, the finite-element method. Unburnt bagasse was the source of the pollutant, we assumed there was no external source. Moreover, there was no other source of air since air was pumped in the chimney by induced drought fan(IDF). Our model relied heavily on the concept of diffusion. Therefore, we used a Two-dimensional diffusion equation[1] to introduce finite element conepts

$$\frac{\partial}{\partial x}(D_h\frac{\partial C}{\partial x})+\frac{\partial}{\partial y}(D_v\frac{\partial C}{\partial y})=f(x,y)\in A \tag{3.1}$$

First,the domain A was separated by imaginary lines/ surfaces into a number of elements(triangles).D_h was the dispersion coefficient along the horizontal D_v was dispersion coefficient along vertical in square units per second. The boundary condition communicating the problem definition to (3.1) was

$$c=c_0 \tag{3.2}$$

where, C refers to concentration, c the boundary/inital value concentration

The elements were assumed interconnected at a discrete number of nodal points on their boundaries[1].The values of c at these nodal points would be the basic unknown parameters. Zienkiewicz [34] discusses the introduction of parameters other than the nodal ones. The direction of wind flow was along the chimney. Induced drought fan(IDF) pump flue gas through the chimney into the atmosphere. The solution to (3.1) can be found analytically by integrating (3.1) directly twice and determine the constants of integration by imposing the boundary conditions (3.2) of any known value.

This solution was independent of an x-axis origin shift; hence, for $L = b - a$ the domain span, and setting $c = 0$ in (3.1) and (3.2), this solution can be verified to be

$$C(x) = \frac{sL^2}{2k}[1 - (\frac{x}{L})^2] + \frac{qL}{K}(1 - \frac{x}{L}) \tag{3.3}$$

We can use (3.3) to compare approximate solutions obtained from the finite-element procedure. Fundamental theories leading to construction of approximate solutions to (3.1) to (3.3) are the Rayleigh-Ritz variational procedure and the Bubnov- Galerkin(B-G)[1] weak statement. In addition, we have the weighted-residual methods, which include collocation, finite-volume, and least-square, each of which constitute a subset within the weak-statement formulation. Notwithstanding which procedure one chooses, the fundamental step is recognition that one seeks an approximate solution for the concentration of the pollutant,C(x). The classical calculus procedure is to determine a Taylor series of known functions of x, each multiplied by unknown constant to be determined according to certain criterion. Hence

$$C^N(x) = \alpha_1 Q_1(x) + \alpha_2 Q_2(x) + \alpha_3 Q_3(x) + \dots\dots\dots + \alpha_N Q_N(x) \qquad (3.4)$$

Is an appropriate expression for an approximation. In (3.3), the α_i for $1 \leq i \leq N$ are unknown arbitrary constants, the $Q_i(x)$ are known functions of x, and superscript on C^N denotes there were N terms in the approximation where;$\alpha_1 = \frac{sL^2}{2K}$, $Q_1(x) = (1 - (\frac{x}{L})^2)$, $\alpha_2 = \frac{qL}{K}$,$Q_2(x) = 1 - \frac{x}{L}$. Unfortunately, the exact solution is never known for the real world problems of ultimate interest. However, as stated in (3.4), any approximation to (one-dimensional) solution can certainly be expressed in the form

$$C^N = \sum_{i=1}^{N} a_i Q_i(x) \qquad (3.5)$$

Our major challenge was obvious, i.e., find or define a suitably general set of functions $1 \leq i \leq N$, such that(3.4) generates a good approximation. Approximating the solution to (3.1) to (3.3) with the series expression(3.5) generally would not coincide with the exact solution. Consequently, an error was created which was simply the difference between the Exact solution and the approximate solution.

3.1 The Finite Element Basis

In evaluating the Galerkin weak statement, one is required to specifically define the trial function set $Q_i(x)$, $1 \leq i \leq N$. We can be very precise about $Q_i(x)$ if knowledge exists about the correct answer. However, practical problems are characterized by a lack of such knowledge. Therefore, we need to select a set of general - purpose functions of Q(x) that will admit versatility. For example, we may recall using Fourier series in calculus to represent a function. Here, the typical term is $Sin(n\pi x)$, and then (3.6) would become;

$$C^N(x) = \sum_{n=1}^{N} a_n Q_n(x) = \sum_{n=1}^{N} a_n sin(n\pi x)/L \tag{3.6}$$

where by convention the summation index is now n rather than i. An alternative selection approach of Q_i draws from interpolation theory. The first estimate might be to use a simple power series in x; hence(3.4) becomes

$$C^N(x) = \sum_{i=0}^{N} a_i x^i = a_0 + \sum_{x=1}^{N} a_i x^i \tag{3.7}$$

and we have $Q_i \equiv x^i$. The constant a_0 provides for a non-zero left-end boundary condition, all $Q_i(x)$ possess a non-zero first derivative and all products for reasonable N are square integrable on (0,L). Therefore, the above possess the basic mathematical requirements, but it also exhibits a fatal flaw. Specifically, for large N, the power series expansion of (3.7) becomes highly oscillatory between evaluation points. The preference was to restrict the value of N to small integer, and to break up the function to be interpolated into small segments with as many local end points on the interior of (0,L) as needed to achieve the desired accuracy. This is called Lagrange interpolation.

3.1.1 Discretization

The difference of of the discretization Ω^h, the disretization domain of,Ω, the domain, was central of finite element analysis. The one-dimensional development for piecewise linear polynomials fully illustrates the concept. The notation for element domain is Ω_e and the discretization domain is Ω^h. The nodes of this discretization are thus XI,X2,and X3. The linear polynomial that is piecewise continuous on Ω^h, and which is centered and of unit value at X2 was;

$$Q_2 = \{\frac{x - x1}{x2 - x1}, \frac{x2 - x}{x3 - x2} \tag{3.8}$$

Thus, (2.10) is piecewise(linear) trial function Q_2 for i=2 the member centered at node X2.

PIECEWISE POLYNOMIAL FUNCTIONS

To begin the discussion of (pp)-functions, let the internal partition 'I' be given by

$\alpha = x_1 < x_2 < \dots\dots < x_{x+1} = b$

with $h = maxhj = max_1 \leq x_j \leq 1(x_{j+1} - x_j)$

Also let $\{P_j(x) \mid_{j=1}, \dots\dots 1$ be any sequence of polynomials of order $k(degree \leq$

$k-1)$. The corresponding(pp)-function, F(x), of order K is defined by

$$F(x) = P_j, x_j < x < x_{j+1} \tag{3.9}$$

where; x_j are called breakpoints of F. By convention
$F(x) = \{P1(x), x \leq x_1 : P_l(x), x \geq x_{l+1}$ and $Fx_i = P_i(x_i)$(right continuity)
The problem was how to conveniently represent the pp-functions: Let S be the set functions;

$$S = \{\lambda_j(x) \mid_j = 1, ..., L \tag{3.10}$$

The class of functions denoted by ℓ is defined to be the set of all functions f(x) of the

$$f(x) = \sum_{j=1}^{L} \beta_j \lambda_j \tag{3.11}$$

where, β_j's are constants.This class of functions ℓ defined by (3.11) is called a linear function space. This is analogous to a linear vector space, for if vector α_j were substituted for the functions $\lambda_j(x)$ in (3.11),we have the usual definition of element x of a vector space. If the functions λ_j in S were linearly independent, then the S is called a
basis for the space ℓ, to the set of all pp-functions of order K on the partition I. The dimension of this space is

$$dim L_k(I) = K\ell \tag{3.12}$$

let V be the sequence of nonnegative integers V_j, that is, $V = V_j \mid = 2, ..., \ell$, such that

$$jumpx_j = \frac{d^{i-1}}{dx^{i-1}}[f(x)] = 0 \tag{3.13}$$

$,i = 1, ..., V_i, j = 2, ..., \ell$

where,

$$jumpx_j = \frac{d^{i-1}}{dx_{i-1}}[f(x)] = \frac{d^{i-1}}{d^{i-1}}[fx_j^{+1}] - \frac{d^{i-1}}{dx^{i-1}}[fx_j^{-1}] \tag{3.14}$$

The dimension of this space ℓ_k^v is

$$dim\ell_k^v(I) = \sum_{J=1}^{\ell}(K - V_j) \tag{3.15}$$

where, $V_1 = 0$.

When using ℓ_k^v as an approximating space for solving differential equations by finite-element methods, we would use variables continuity through out the interval. The simplest space is $\ell_2^1(I)$, the space of piecewise linear functions. A basis for this space consists of straight line segments(degree=1) with discontinuous derivatives at the breakpoints(v=1). Our domain of study was the Boiler-Chimney of height, Hm. We assumed the chimney was a line segment with the actual height of 30.98m. Also, we assumed the wind was pumped only in one direction by the induced drought fan(ID-fan), that there was neither external source of wind nor pollutant. The pollutant was from unburnt bagasse brought as a result of insufficient combustion and wet bagasse.

Linear Basis Functions

This basis can be written as;

$w_1 = \frac{x_2 - x}{x_2 - x_1}$, for $x_1 \leq x \leq x_2$;

$w_1 = 0$, for $x \geq x_2$

$w_j \frac{x - x_{j-1}}{x_j - x_{j-1}}$, for $x_{x_j - x_{j-1}}$, for $x_{j-1} \leq x \leq x_j$

$w_j = \frac{x_{j+1} - x}{x_{j+1} - x_j}$, for $x_j \leq x \leq x_{j+1}$

$w_j = 0$, for $x_{j-1} \leq x, x \geq x_{j+1}$

In many physical situations, we are concerned with the diffusion or flow or advection of some quantity such as heat, mass, pollutants and chemicals, e.t.c. In such problems the rate of transfer per unit area, q, can be written in terms of its cartesian components as

$$q^T = [q_x, q_y, q_z] \tag{3.16}$$

If the rate at which the relevant quantity was generated(or removed) per unit volume was Q, then for steady-state flow the balance or continuity requirement gives

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} + Q = 0 \tag{3.17}$$

Introducing gradient operator

$$\nabla = \{\frac{\partial}{\partial x}; \frac{\partial}{\partial y}; \frac{\partial}{\partial z}\}^T \tag{3.18}$$

We can write the above as

$$\nabla^T q + Q = 0 \tag{3.19}$$

Generally the rates of flow will be related to gradients of some potential quantity Q. This may be the temperature in case of advection-diffusion, e.t,c.. A very general linear relationship will be of the form

$$q = \{q_x, q_y, q_z\}^T = -K\{\frac{\partial Q}{\partial x}, \frac{\partial Q}{\partial y}, \frac{\partial Q}{\partial z}\}^T \tag{3.20}$$

3.1.2 Interpolating Function

Successfully solving a problem by the finite element method rests primarily on choosing a trial function which takes the form of a polynomial with a number of undetermined coefficients(parameters). The parameters must be determined in the vicinity of an element. According to the degree of the interpolating functions, finite elements are classified into three groups as follows (a)Simplex Elements (b) Complex Elements (c) Multiplex Element.

3.1.3 The Simplex Element

Has an approximating polynomial which is composed of constant and linear terms example

$\theta(x) = \alpha_1 + \alpha_2 x + \alpha_3 y$

3.1.4 The Complex Element

Has an approximating polynomial which is composed of constant, linear and higher-order terms. Example,
$\theta(x) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 y^2$

3.1.5 The Multiplex Element

Is an approximating polynomial which has a constant,linear , higher order and multiple terms. Example,
$\Theta(x) = \theta(x) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 y^2 + \alpha_6 xy$
Finally, our model used mathematica to determine the internal nodal concentration of the pollutant in the last subsection of this chapter. The models that we used are:
For Concentration(C) of the pollutant,Fly-ash

where,N_i, N_j and N_k are the shape functions and i,j and k are index for nodes. C_i, C_j and C_k are known values. However, C is unknown and to be determined. The interpolating linear polynomial governing equation(3.22) is

$$C = \alpha_1 + \alpha_2 x + \alpha_3 y \tag{3.21}$$

with the nodal conditions

The one-dimensional simplex element is simply a line segment with length or height L or H which act as the domain or range along which the parameter C is approximated within and between the nodes at i and j respectively. The parameter C is approximated by linear polynomial function

$$C = \alpha_1 + \alpha_2 x \tag{3.22}$$

The unknown coefficients,α_1 and α_2 can be determined by using the nodal conditions

$$C = C_i \tag{3.23}$$

at; $x = x_i$

and;

$$C = C_j \tag{3.24}$$

at $x = x_j$. This result in the following pair of equations

$$C_i = \alpha_1 + \alpha_2 x_i \tag{3.25}$$

$$C_j = \alpha_1 + \alpha_2 x_j \tag{3.26}$$

The solutions for the parameters are

$$\alpha = \frac{1}{L}(C_i x_j - C_j x_i) \tag{3.27}$$

$$\alpha = \frac{1}{L}(C_j - C_i) \tag{3.28}$$

where; $L = x_j - x_i$ is the or domain

$$C_i = \alpha_1 + \alpha_2 x_i \tag{3.29}$$

$$C_j = \alpha_1 + \alpha_2 x_j \tag{3.30}$$

Let; $\alpha_1 = C_i + \alpha_2 x_i$

By substitutions

$\Rightarrow (C_i - \alpha_2 x_j) + \alpha_2 x_j$

$C_j = (C_i - \alpha_2 x_i) + \alpha_2 x_j$

$C_j = \alpha_2(x_j - x_i)$

$C_j - C_i = \alpha_2(x_j - x_i)$

$\Rightarrow \alpha_2 = \frac{C_j - C_i}{x_j - x_i} = \frac{1}{L}(C_j - C_i)$

Now, $\alpha_1 = \frac{C_i x_j - C_j x_i}{L}$

and, $\alpha_2 = \frac{1}{L}(C_j - C_i)$

Thus, we likewise substitute α_1 and α_2 in our linear polynomial function

$C = \alpha_1 + \alpha_2 x$

$C = \frac{C_i x_j - C_j x_i}{L} + \frac{x}{L}(C_j - C_i)$

$= \frac{1}{L}[C_i x_j - C_j x_i + C_j x - C_i x]$

$= \frac{1}{L}(x_j - x)C_i + \frac{1}{L}(x - x_i)C_j$

Let , $N_1 = \frac{1}{L}(x_j - x)$ and, $N_2 = \frac{1}{L}(x - x_i)$

Our model for the concentration of air pollutant concentration is thus given by;

$$C = N_1 C_s + N_2 C_T \tag{3.31}$$

Where, N_1 and N_2 , are called shape functions of the element. The shape functions always lie between 0 and 1 such that; $0 \leq N_1 \leq 1$, and, $0 \leq N_2 \leq 1$. C_s, is the concentration at the source, C_T, is terminal concentration. Consequently, our concentration gradient is defined as follows

$$\frac{\partial C}{\partial x} = \frac{\partial N_1}{\partial x}(C_i) + \frac{\partial N_2}{\partial x}(C_j) = \frac{1}{L}[C_j - C_i] \tag{3.32}$$

where, C is unknown fly-ash Concentration , C_i , C_j, and C_k are known fly-ash concentration at nodal ends. To illustrate this we consider the following example. Find the fly-ash concentration within the chimney at x = 6cm given that concentration at x_i = 2cm and x_j=8cm, are C_i = 30ppm and C_j = 24ppm respectively. The appropriate concentration at x =6cm, can be obtained by the following shape function

$$C = N_i C_i + N_j C_j \tag{3.33}$$

$N_i = \frac{1}{6}(x_j - x)$, $N_j = \frac{1}{6}(x - x_i)$, $L = x_j - x_i$ = 8-2=6cm, C_i = 30ppm and C_j = 24ppm

$N_i = \frac{1}{6}(8 - 6)$= 0.33333, $N_j = \frac{1}{6}(6 - 2) = 0.6667$
$C = .3333(30) + 0.6667(24) = 26ppm$

Since air pollution at Sony-Sugar is a product of temperature and pressure, we also developed models for both. Our Temperature model or shape function for temperature is

$$T = N_i T_i + N_j T_j + N_k T_k \tag{3.34}$$

where, T is unknown temperature , T_i , T_j, and T_k are known temperature at nodal ends. To illustrate this we consider the following example.

Find the temperature of the rod at x = 6cm given that temperature at x_i = 2cm and x_j=8cm, are $T_i = 100^0C$ and $T_j = 60^0C$ respectively. The appropriate temperature at x =6cm, can be obtained by the following shape function

$$T = N_iT_i + N_jT_j \tag{3.35}$$

$N_i = \frac{1}{6}(x_j - x)$, $N_j = \frac{1}{6}(x - x_i)$, $L = x_j - x_i$ = 8-2=6cm, $T_i = 100^0C$ and $T_i = 60^0C$

$N_i = \frac{1}{6}(8-6)$= 0.33333, $N_j = \frac{1}{6}(6-2) = 0.6667$
$T = .3333(100) + 0.6667(60) = 73.3^0C$
Similarly, our shape function or model for pressure in this study is given by

$$P = N_iP_i + N_jP_j + N_kP_k \tag{3.36}$$

where, P is unknown pressure , P_i , P_j, and P_k are known pressure at nodal ends. To illustrate this, we consider the following example. Find the pressure of the chimney at x = 6cm given that pressure at x_i = 2cm and x_j=8cm, are $P_i = 22.0bar$ and $P_j = 21.0bar$ respectively. The appropriate pressure at x =6cm, can be obtained by the following shape function

$$P = N_iP_i + N_jP_j \tag{3.37}$$

$N_i = \frac{1}{6}(x_j - x)$, $N_j = \frac{1}{6}(x - x_i)$, $L = x_j - x_i =$ 8-2=6cm, $P_i = 22.0$ and $P_j = 21.0bar$

$N_i = \frac{1}{6}(8-6)$= 0.33333, $N_j = \frac{1}{6}(6-2) = 0.6667$

$T = .3333(22.0) + 0.6667(21.0) = 21.3bar$

Since in our study a two dimensional finite element was used determine concentration of the pollutant, its trial function is given by the following linear polynomial function;

$Q = \alpha_1 + \alpha_2 x + \alpha_3 y,$

thus, the nodal conditions are

$Q = Q_i$ at $x = x_i; y = y_i$

$Q = Q_j$ at $x = x_j; y = y_j$

$Q = Q_k$ at $x = x_k; y = y_k$

Now, we have a set of algebraic equations

$Q_i = \alpha_1 + \alpha_2 x_i + \alpha_3 y_i$

$Q_j = \alpha_1 + \alpha_2 x_j + \alpha_3 y_j$

$Q_k = \alpha_1 + \alpha_2 x_k + \alpha_3 y_k$

In matrix form, $AV = g$

$$\begin{pmatrix} 1 & x_i & y_i \\ 1 & x_i & y_j \\ 1 & x_k & y_k \end{pmatrix} \begin{pmatrix} \alpha_i \\ \alpha_j \\ \alpha_k \end{pmatrix} = \begin{pmatrix} Q_i \\ Q_j \\ Q_k \end{pmatrix}$$

Where,

$Q = NiQ_i + N_jQ_j + N_kQ_k.$

The concentration gradient is given by;

$\frac{\partial Q}{\partial x} = \frac{\partial N_1}{\partial x}Q_i + \frac{\partial N_2}{\partial x}Q_j + \frac{\partial N_3}{\partial x}Q_k$

$\frac{\partial Q}{\partial y} = \frac{\partial N_1}{\partial y} Q_i + \frac{\partial N_2}{\partial y} Q_j + \frac{\partial N_3}{\partial y} Q_k$

The element we are dealing with is a 2-D triangular element whose area is denoted by A. N_i, N_j and N_k are the shape functions and are functions of x and y respectively.

$N_1 = \frac{1}{2A}(a_i + b_i x + c_i y)$

$a_i = x_j y_k - x_k y_j$

$b_i = y_j - y_k$

$c_i = x_k - x_j$;

also,

$N_2 = \frac{1}{2A}(a_j + b_j x + c_j y)$

$a_j = x_k y_i - x_i y_k$

$b_j = y_k - y_i$

$c_j = x_i - x_k$;

$N_3 = \frac{1}{2A}(a_k + b_k x + c_k y)$

$a_k = x_i y_j - x_j y_i$

$b_k = y_i - y_j$

$c_k = x_j - x_i$

Finally, we substitute the above parameters into;

$Q = N_1 Q_i + N_2 Q_j + N_3 Q_k|_{7,12}$, and

$\rightarrow x = 7m$

$\rightarrow x = 12m$

we let Q=C

$\Rightarrow Q = N_1 Q_i + N_2 Q_j + N_3 Q_k|_{7,12}$

$a_i = -25, b_i = 5, c_i = 0$

$N_1 = \frac{1}{25}(a_i + b_i x + c_i y)(C_i)$

$N_1 = \frac{1}{25}(-25 + 5(7) + 0(12)(20))$

$N_1 Q_i = 8mg/m^3$

$N_1 = \frac{1}{25}(a_j + b_j x + c_j y)(C_j)$

$N_2 = \frac{1}{25}(-50 + 0(7) + 5(12)(50)$

$N_2 Q_j = 20mg/m^3$

$N_3 = \frac{1}{25}(a_k + b_k x + c_k y)(C_k)$

$N_3 = \frac{1}{25}(100 - 5(7) - 5(12)(30)$

$N_3 Q_k = 6mg/m^3$

Therefore

$Q = N_1 Q_i + N_2 Q_j + N_3 Q_k$

$Q = 8 + 20 + 6 = 34mg/m^3$

thus, the concentration of the pollutant over the surface at (7,12) is $34mg/m^3$. In this study, two dimensional finite element scheme is used to determine the nodal concentration of air pollutant, the fly-ash within the the boiler chimney. We assumed the chimney cross-section area is a polygonal in shape. This polygonal shape is irregular, thus the elements in it are not all equal in size and area as well. Our elements are triangular in shape with one interior vertex and the rest outside. In this connection,i,j and k are the vertices indexes, and the elements are denoted by . Basically, the polynomial linear interpolation equation for our model for determination of the nodal concentration is developed by

$$C = \alpha_1 + \alpha_2 x + \alpha_3 y \tag{3.38}$$

where, α_1 ,α_2, α_3 are undetermined constants, x and y variables.

3.1.6 Discretization of the Two-Dimensional Element

To develop a two-dimension simplex element, many triangles or quadrilaterals are usually employed for finite element mesh. Just as the one dimensional case, the next step is to develop systems of algebraic equations to approximate the solution for the two-dimensional finite element. In this connection we have;

$$C(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y \tag{3.39}$$

where C(x,y) is the dependent variable, and x and y the independent variables. This is an illustration, and the function must pass through the values of C(x,y) at triangle's nodes(x_1, y_1),(x_2, y_2) and (x_3, y_3). Therefore,

$C_1(x, y) = a_1 + a_2 x_1 + a_3 y_1$

$C_2(x, y) = a_1 + a_2 x_2 + a_3 y_2$

$C_3(x, y) = a_1 + a_2 x_3 + a_3 y_3$

$$\begin{pmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}$$

3.1.7 Interpolating Polynomial For a Discretized Region

Next, we consider the two-dimensional simplex element. This is a triangular element with three nodes.The interpolating polynomial equation(3.40) with nodal conditions

$C = C_i$ at $x = x_i$; $y = y_i$

$C = C_j$ at $x = x_j$; $y = y_j$

$C = C_k$ at $x = x_k$; $y = y_k$

Now, we have a set of algebraic equations

$C_i = \alpha_1 + \alpha_2 x_i + \alpha_3 y_i$

$C_j = \alpha_1 + \alpha_2 x_j + \alpha_3 y_j$

$C_k = \alpha_1 + \alpha_2 x_k + \alpha_3 y_k$

$$\begin{pmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} C_i \\ C_j \\ C_k \end{pmatrix}$$

Solving for α_1, α_2, and α_3, substituting in (3.40), the model equation and rearranging the terms, equation (2.42) can be expressed as

$$C = N_i^e C_i + N_j^e C_j + N_k^e C_k \tag{3.40}$$

which is the model used to determine the internal concentration of air pollutant at Sony-Sugar Company in Kenya. We used mathematica 5 to solve the algebraic linear equations forming the matrix. In (3.42), N_i^e, N_j^e, and N_k^e are called the shape functions, which depend on both x and y, where e = 1,2,3...........,r

$N_i^e = \frac{1}{2A}[a_i^e + b_i^e x + c_i^e y]$

the unknown constants a_i^e, b_i^e and c_i^e are determined as follows

$a_i^e = X_j Y_k - X_k Y_j$

$b_i^e = Y_j - Y_k$

$c_i^e = X_k - X_j$

$N_j^e = \frac{1}{2A}[a_j^e + b_j^e x + c_j^e y]$

the unknowns are ;

$a_j^e = X_k Y_i - X_i Y_k$

$b_j^e = Y_k - Y_i$

$c_i^e = X_i - X_k$

$N_k^e = \frac{1}{2A}[a_k^e + b_k^e x + c_k^e y]$

the unknown constants are;

$a_k^e = X_i Y_j - X_j Y_i$

$b_k^e = Y_i - Y_j$

$c_k^e = X_j - X_i$

The governing local matrix for the algebraic equations is

$$A = \frac{1}{2}\begin{pmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{pmatrix}$$

i.e A is the area of the triangle

Note; the triangles do not necessarily possess the same area. The global matrix of the above phenomena is given by;

$$A_{c1} = \begin{pmatrix} N_1^1 & N_2^1 & N_3^1 & 0 & 0 & 0 \\ 0 & N_2^2 & N_3^2 & N_4^2 & 0 & 0 \\ 0 & 0 & N_3^3 & N_4^3 & N_5^3 & 0 \\ 0 & 0 & N_3^4 & 0 & N_5^4 & N_6^4 \\ N_1^5 & 0 & N_3^5 & 0 & 0 & N_6^5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

which is the general model used in chapter four to analyze the pollutant concentration at Sony factory. Similarly, an array model for smaller triangular elements constructed. We the chimney assumed the chimney cross-section area is a polygon of ten sides. The cross-section is discretized into triangular elements, thus a set of linear algebraic equations is generated in the form of a matrix;

$$AU = C \tag{3.41}$$

Where, A is the coefficient matrix of the shape functions; U, is the unknown column vector matrix to be determined when some known values are given; C, is the vector of some known values, in our case the data obtained from the boiler log daily operations report of Sony-Sugar factory.

$$\begin{pmatrix} .33 & .43 & .31 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .36 & .31 & .33 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .23 & .38 & .38 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .37 & 0 & .35 & .29 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .30 & 0 & 0 & .58 & .37 & 0 & 0 & 0 & 0 \\ 0 & 0 & .34 & 0 & 0 & 0 & .38 & .31 & 0 & 0 & 0 \\ 0 & 0 & .33 & 0 & 0 & 0 & 0 & .33 & .35 & 0 & 0 \\ 0 & 0 & .41 & 0 & 0 & 0 & 0 & 0 & .44 & .14 & 0 \\ 0 & 0 & -1.3 & 0 & 0 & 0 & 0 & 0 & 0 & .31 & 2.41 \\ .38 & 0 & .22 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .41 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} C_1^1 \\ C_2^2 \\ C_3^3 \\ C_4^4 \\ C_5^5 \\ C_6^6 \\ C_7^7 \\ C_8^8 \\ C_9^9 \\ C_{10}^{10} \\ C_{11}^{11} \end{pmatrix} =$$

Table 3.1: Con.of fly-ash

35.51	29.38	2.10	-35.8	34.53	24.61	-40.08	47.08	5.06	-22.06	2.56

$$\begin{pmatrix} 25 \\ 0 \\ 0 \\ 0 \\ 18 \\ 0 \\ 0 \\ 0 \\ 15 \\ 0 \\ 0 \end{pmatrix}$$

The above matrix was obtained from the 2-D finite element equation(3.42) which we develop to determine pollutant concentration.The table below has output values obtained by solving the matrix using MATHEMATICA.

The superscripts denote the element,e i.e. e=1,2,3,..................,e

Chapter 4

Results And Discussions

4.1 Determination of Concentration of Air Pollutant

In this chapter we used our models in chapter three to determine the pollutant concentration. Our models are;3.24,3.38 and 3.40 for the Pollutant, Temperature and Pressure respectively. Tables and Graphs generated by these models are also illustrated.To achieve this we introduced the concepts by illustration of a few models governing solution of equations such as the;(a) The Iterative methods, (b) Jacobi Method and Taylor polynomial function.

(a) **Iterative Methods**. An iterative method for solving equations is one in which a first approximation is used to calculate a second approximation which in turn is used to calculate a third and so on. The iterative procedure is said to be convergent if the differences between exact solution of the successive approximations tends to zero as the number iterations increase. In general, the exact solution is never obtained in a finite number of steps, but this does not matter. Notwithstanding, the important thing is that the successive iteratives converge fairly rapidly to values that are correct to specific accuracy. (b) **Jacobi Method** . Denote the first approximation by g_i^1, the second by g_i^2,e.t.c. and assume that n of them have been calculated, i.e.g_i^n is known for $i = I(I)m$.. Then the Jacobi iterative method expresses the $(n+1)^{th}$ iterative values exclusively in terms of the nth iterative values. In general case, for m equations, the systems of linear algebraic equations i.e $Ag = b$

$$g_i^{n+1} = \frac{1}{a_{ii}}\{b_i - \sum_{j=1}^{i-1} a_{ii}g_j^{n+1} - \sum_{j+1}^{n} a_{ii}g_j^n, i = I(I)\} \tag{4.1}$$

4.1.1 Calculations

$$N_1 = \frac{1}{L}(x_j - x) + \frac{1}{L}(x - x_i) \tag{4.2}$$

where

Note: N_1 and N_2 are dependent variables in x , and x independent variable of C, N_1, N_2

We shall illustrate in a few steps the validity of our model and give the whole results in the tables later.

The model is

$$C = N_1 C_S + N_2 C_T \tag{4.3}$$

at x = 2m

$C = \frac{1}{28}(30-2)C_S + \frac{1}{28}(2-2)C_T = \frac{28}{28}(23) + 0 = 23ppm$

at x = 4m; $C = \frac{1}{28}(30-4)(23) + \frac{1}{28}(4-2)(15) = \frac{26}{28}(23) + \frac{2}{28}(15) = 22.43ppm$

at x =6m;$C = \frac{1}{28}(30-6)(23)+\frac{1}{28}(6-2)(15) = \frac{24}{28}(23)+\frac{4}{28}(15) = 21.86ppm$

at x = 8m; $C = \frac{1}{28}(30-8)(23) + \frac{1}{28}(8-2)(15) = \frac{22}{28}(23) + \frac{6}{28}(15) = 21.29ppm$

at x = 10m; $C = \frac{1}{28}(30-10)(23) + \frac{1}{28}(10-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) = 20.71ppm$

at x=12m;$C = \frac{1}{28}(30-12)(23) + \frac{1}{28}(12-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) = 20.14ppm$

at x=14m;$C = \frac{1}{28}(30-14)(23) + \frac{1}{28}(14-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) = 19.57ppm$

at x=16m;$C = \frac{1}{28}(30-16)(23)+\frac{1}{28}(16-2)(15) = \frac{20}{28}(23)+\frac{8}{28}(15) = 19ppm$

at x=18m;$C = \frac{1}{28}(30-18)(23) + \frac{1}{28}(18-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) =$ $18.43ppm$

at x=20m; $C = \frac{1}{28}(30-20)(23) + \frac{1}{28}(20-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) =$ $17.86ppm$

at x=22m; $C = \frac{1}{28}(30-22)(23) + \frac{1}{28}(22-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) =$ $17.29ppm$

at x= 24m; $C = \frac{1}{28}(30-24)(23) + \frac{1}{28}(24-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15)$ $= 16.71ppm$

at x=26m; $C = \frac{1}{28}(30-26)(23) + \frac{1}{28}(26-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) =$ $16.14ppm$

at x=28m; $C = \frac{1}{28}(30-28)(23) + \frac{1}{28}(28-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) =$ $15.57ppm$

at x= 30m;$C = \frac{1}{28}(30-30)(23) + \frac{1}{28}(30-2)(15) = \frac{20}{28}(23) + \frac{8}{28}(15) =$ $15.00ppm$

and e.t.c.

Table(3.1) below is a combined table for temperature(in Degrees celsius), pressure(in Bar) and fly-ash(in ppm i.e parts per million). The source of this data is: Boiler log daily operations report of Sony-Sugar Company(SONY),Kenya.

Note:In fig(4.1) the horizontal axis denotes the height of the chimney and the vertical axis i.e y-axis represents pollutant values in parts per million(ppm).

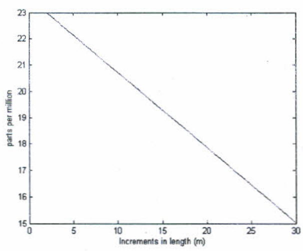

Figure 4.1: Fly-ash concentration within the chimney

The above graph illustrated a smooth linear relation of the fly-ash within the chimney. A good show of well distribution within the chimney.

At Sony Sugar the boiler operating Temperature is between 330^0C to 380^0C. The table below the temperature distribution sufficient to operate the boiler.

Our graph in fig.(4.2) below showed a smooth linear relation, an indication well distribution of Temperature within the chimney.

In fig.(4.3) above, Pressure variation within the chimney is not Uniform.The graph is not smooth as expected.

$$P_g = \rho hg + Atmosphericpressur \tag{4.4}$$

Table 4.1: Combined table

F.A	TEMP	PRESS.
15	330	22.0
15.4	332	22.06
15.8	334	22.12
16.2	336	22.18
16.6	338	22.24
17.0	340	22.30
17.4	342	22.36
17.8	344	22.42
18.2	346	22.48
18.6	348	22.54
19.0	350	22.60
19.4	352	22.66
19.8	354	22.72
20.2	356	22.78
20.6	360	22.84
21.0	362	22.90
21.4	364	22.96
21.8	366	23.02
22.2	368	23.08
22.6	370	23.14
23.0	372	23.20
23.4	374	23.26
23.8	376	24.32
24.2	378	24.38
24.6	380	24.44
25.0	382	23.50

Table 4.2: Fly-Ash

Increaments $\triangle x = 2m$	Conc.o. fly-ash
2	23
4	22.43
6	21.86
8	21.29
10	20.71
12	20.14
14	19.57
16	19.00
18	18.43
20	17.86
22	17.29
24	16.71
26	16.14
28	15.57
30	15.00

Table 4.3: Temperature in degrees Celsius

Increaments $\triangle x = 2m$	Temp.in degrees celsius
2	380
4	376.4
6	372.9
8	369.3
10	365.7
12	362.1
14	358.6
16	355.0
16	351.4
20	347.9
22	344.3
24	340.7
26	337.1
28	333.6
30	330.0

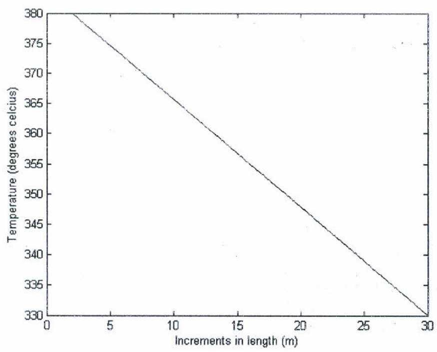

Figure 4.2: Temperature variation within the chimney

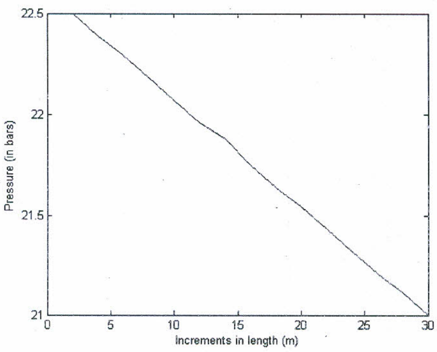

Figure 4.3: Pressure variation within the chimney

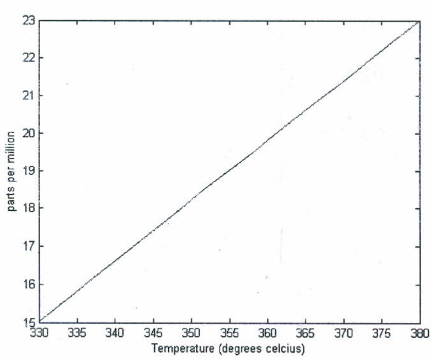

Figure 4.4: Fly-ash concentration versus Temperature variation

Table 4.4: Pressure in Bars

Increaments $\triangle x = 2m$	Press.in bars
2	22.5
4	22.39
6	22.29
8	22.18
10	22.07
12	21.96
14	21.88
16	21.75
18	21.64
20	21.54
22	21.43
24	21.32
26	21.21
28	21.11
30	21.00

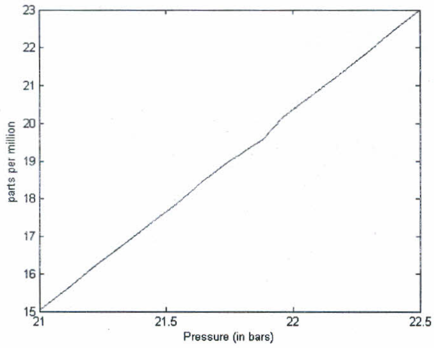

Figure 4.5: Fly-ash concentration versus Pressure variation

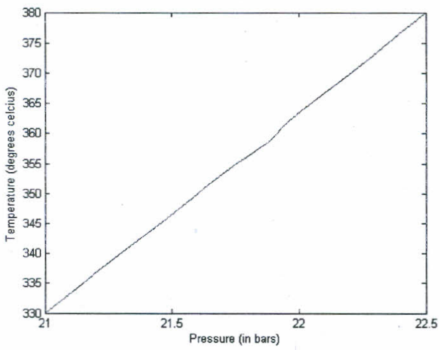

Figure 4.6: Temperature - Pressure variation

In our general observations the only variables which showed good linear relationships were of figure (4.4),the Fly-Ash Vs Temperature graph. Flay-Ash Vs Pressure graph and Temperature Vs Pressure graph were a good indication of causes of this high production of the pollutant,"FLY-ASH". To conclude, tables 4.1,4.2,4.3 and 4.4 respectively, were generated using our mathematical model to supply as with data for our analysis.

Chapter 5

Application of the Model And Analysis

5.1 Application of Models

In Thongmoon[28] conducted numerical experiment of concentration of air pollutant using a three dimensional advection - diffusion equation. In their research they used Finite-Difference scheme to determine the concentration of air pollutants within the street tunnel. In this study we developed 2-D finite-element equations as indicated earlier in equations (2.38) ,(2.40) and (2.44) respectively.The shape functions governing each element are denoted by superscript (e) meaning element i.e. $e = 1, 2, 3, ..., k$ and the subscript i,j and k denote nodes as stated earlier.

Element(1) matrix; $E_1 = \begin{pmatrix} 1 & 3 & 7 \\ 1 & 7 & 2 \\ 1 & 5 & 6 \end{pmatrix}$ Area;$2A = 18 \mid 5, 3$

for model[1],

$a_1^1 = X_2Y_3 - X_3Y_2 = 42 - 10 = 32;$

$b_1^1 = Y_2 - Y_3 = 2 - 6 = -4$

$N_1^1 = \frac{1}{18}[32 - 20 - 6] = 0.333;$

for node [2],

$N_2^1 = \frac{1}{18}[a_2^1 + b_2^1 x + c_2^1 y]$

$a_2^1 = 10 - 18 = -8$

$b_2^1 = 6 - 4 = 2$

$c_2^1 = 3 - 5 = -2$

$N_2^1 = \frac{1}{18}[-8 + 20 - 6] = 0.333$

for node[3];$N_3^1 = \frac{1}{18}[a_3^1 + b_3^1 x + c_3^1 y] = 0.222$

Element[2] Matrix;

$$E_2 = \begin{pmatrix} 1 & 7 & 2 \\ 1 & 8 & 6 \\ 1 & 5 & 6 \end{pmatrix} \text{ Area;2A=12}$$

Node [2] shape function

$N_2^2 = \frac{1}{18}[a_2^2 + b_2^2 x + c_2^2 y]$

$a_2^2 = 48 - 30 = 18$

$b_2^2 = 6 - 6 = 0$

$c_2^2 = 5 - 8 = -3$

$N_2^2 = \frac{1}{18}[18 + 0 - 15] = 0.25$

$N_3^2 = \frac{1}{12}[a_3^2 + b_3^2 x + c_3^2 y] = 0.25$

$N_4^2 = \frac{1}{12}[a_4^2 + b_4^2 x + c_4^2 y]$

$a_4^2 = 10 - 42 = -32$

$b_4^2 = 6 - 2 = 4$

$c_4^2 = 7 - 5 = 2$

$N_4^2 = \frac{1}{12}[-32 + 28 + 10] = 0.5$

Element three[3] Matrix;

$$E_3 = \begin{pmatrix} 1 & 8 & 6 \\ 1 & 5 & 9 \\ 1 & 5 & 6 \end{pmatrix}$$

Area; 2A=9

$N_3^3 = \frac{1}{9}[a_3^3 + b_3^3 x + c_3^3 y]$

$a_3^3 = X_4 Y_5 - X_5 Y_4 = 9(8) - 5(6) = 42$

$b_3^3 = 6 - 9 = -3$

$c_3^3 = 5 - 8 = -3$

$N_3^3 = \frac{1}{9}[42 - 39] = 0.3333$

$N_4^3 = \frac{1}{9}[a_4^3 + b_4^3 x + c_4^3 y]$

$a_4^3 = X_5 Y_3 - X_3 Y_5 = 30 - 45 = -15$

$b_4^3 = 9 - 6 = 3$

$c_4^3 = 5 - 5 = 0$

$N_4^3 = \frac{1}{9}[-15 + 18 + 0] = 0.3333$

$N_5^3 = \frac{1}{9}[a_5^3 + b_5^3 x + c_5^3 y]$

$a_5^3 = X_4 Y_3 - X_3 Y_4 = 30 - 48 = -18$

$b_5^3 = 6 - 6 = 0$

$c_5^3 = 8 - 5 = 3$

$N_5^3 = \frac{1}{9}[-18 + 0 + 21] = 0.3333$

Element[4] Matrix;

$$E_4 = \begin{pmatrix} 1 & 5 & 6 \\ 1 & 5 & 9 \\ 1 & 2 & 6 \end{pmatrix}$$

Area = 9

$N_3^4 = \frac{1}{9}[a_3^4 + b_3^4 x + c_3^4 y] \mid_{4,7}$

$a_3^4 = X_5Y_6 - X_6Y_5 = 30 - 18 = 12$

$b_3^4 = 9 - 6 = 3$

$c_3^4 = 2 - 5 = -3$

$N_3^4 = \frac{1}{9}[12 + 12 - 21] = 0.333$

$N_5^4 = \frac{1}{9}[a_5^4 + b_5^4 x + c_5^4 y] \mid_{4,7}$

$a_3^4 = X_6Y_3 - X_3Y_6 = 12 - 30 = -18$

$b_3^4 = 6 - 6 = 0$

$c_3^4 = 5 - 2 = 3$

$N_5^4 = \frac{1}{9}[-18 + 0 + 21] = 0.333$

$N_6^4 = \frac{1}{9}[a_6^4 + b_6^4 x + c_6^4 y] \mid_{4,7}$

$a_6^4 = X_3Y_5 - X_5Y_3 = 45 - 30 = 15$

$b_6^4 = 6 - 6 = -3$

$c_6^4 = 5 - 5 = 0$

$N_6^4 = \frac{1}{9}[15 - 12 + 0] \mid_{4,7} = 0.333$ Element[5] Matrix;

$$E_5 = \begin{pmatrix} 1 & 2 & 6 \\ 1 & 3 & 2 \\ 1 & 5 & 6 \end{pmatrix}$$

Area = 12

$N_1^5 = \frac{1}{12}[a_1^5 + b_1^5 x + c_1^5 y] \mid_{4,7}$

$a_1^5 = X_6Y_3 - X_3Y_6 = 12 - 30 = -18 or 18$

$b_1^5 = 6 - 6 = 0$

$c_1^5 = 2 - 5 = -3$

$N_1^5 = \frac{1}{12}[18 - 15] = 0.25$

But , when

$N_3^5 = \frac{1}{12}[a_3^5 + b_3^5 x + c_3^5 y] \mid_{4,7}$

$a_3^5 = X_6 Y_1 - X_1 Y_6 = 4 - 18 = -14$

$b_3^5 = 6 - 2 = 4$

$c_3^5 = 3 - 3 = 1$

$N_3^5 = \frac{1}{12}[-14 + 16 + 5] = 0.583$

$N_6^5 = \frac{1}{12}[a_6^5 + b_6^5 x + c_6^5 y] \mid_{4,7}$

$a_6^5 = X_1 Y_3 - X_3 Y_1 = 18 - 10 = 8$

$b_1^5 = 2 - 6 = -2$

$c_1^5 = 5 - 3 = 2$

$N_1^5 = \frac{1}{12}[8 - 16 + 10] = 0.167$

Since there are only five elements the sixth raw of the matrix is fictitious (i.e are all zeros). Thus the Global matrix for the model equation(2.44) is;

$$\begin{pmatrix} .333 & .333 & .222 & 0 & 0 & 0 \\ 0 & .250 & .250 & 0.5 & 0 & 0 \\ 0 & 0 & .333 & .333 & .333 & 0 \\ 0 & 0 & .333 & 0 & .333 & .333 \\ .25 & 0 & .583 & 0 & 0 & .167 \\ 0 & 0 & & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} C^1 \\ C^2 \\ C^3 \\ C^4 \\ C^5 \\ C^6 \end{pmatrix} = \begin{pmatrix} 25 \\ 0 \\ 0 \\ 0 \\ 15 \\ 0 \end{pmatrix}$$

Table 5.1: Combined table of concentration of pollutant

Fly-1	res.1	Fly-2	res.2	Fly-3	res.3	Fly-4	res.4	Fly-5	res.5
25	43.20	25	59.15	22	54.50	22	51.84	20	48.74
0	23.64	0	3.23	0	-2.09	0	4.32	0	0
0	12.36	18	19.04	18	20.48	16	14.86	16	15.82
0	-18.00	0	-11.14	0	-9.20	0	-9.59	0	-8.30
15	5.64	15	46.15	15	42.77	12	42.77	12	40.52
0	-18.00	0	-65.19		0	-63.25	-57.64	0	-56.35
0	0	0	0	0	0	0	0	0	0

5.2 Analysis

In this thesis, we analyzed our work using the product moment correlation coefficient[29]. Product moment correlation gives an indication of the strength of linear relationship between two variables, in our case the variables temperature, pressure and the pollutant, fly-ash.

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} X \sqrt{n\sum y^2 - (\sum y)^2}} \tag{5.1}$$

As stated earlier in this study, our data is obtained from boiler log book daily operations report. The table below gives the calculated values;

$$r = \frac{631600 - 629835}{\sqrt{121025} \times \sqrt{1549}} \tag{5.2}$$

$r = \frac{1765}{1369}$

Table 5.2: Fly -Temp

WKs	X	Yppm	XY	X^2	Y^2
1	300	15	4500	90000	0
2	320	23	7360	102400	0
3	310	25	7750	96100	0
4	340	18	6120	115600	0
5	350	20	7000	122500	0
6	305	14	4270	93025	0
7	280	16	4480	78400	0
8	380	20	7600	144400	0
9	250	22	5500	62500	0
10	330	26	8580	108900	0
$\sum$	3165	199	63160	1013625	4115

$r = 1.29$

1.29 indicates a positive correlation between temperature and the pollutant Fly-ash but the ideal correlation should be one(1) for a stronger and positive correlation. The deviation of ± 0.29 could be attributed to poor handling of cane, obsolete equipments,wet bagasse and generally human error.

Where; X is for temperature in degree Celsius, and Y is for fly-ash in parts per million (ppm)

Table 5.3: Fly -Temp

WKs	Pr(X)	Yppm	XY	X^2	Y^2
1	18	15	270	324	225
2	19	16	304	361	256
3	20	18	360	400	324
4	21	14	294	441	196
5	21	20	420	441	400
6	22	20	440	484	400
7	22	23	506	484	529
8	22	25	550	484	625
9	23	22	506	529	484
10	23	26	598	529	676
$\sum$	211	199	4248	4477	4115

Table 4.3 shows the relationship between Fly-ash and Pressure

$$r = \frac{42480 - 41989}{\sqrt{44770 - 4454} \times \sqrt{41150 - 39601}} \tag{5.3}$$

$$r = \frac{491}{\sqrt{249} \times \sqrt{1549}} \tag{5.4}$$

$r = 0.79$

The value of r is 0.79, this indicates a positive correlation between pressure and Fly-ash. This implies that 79 percent of fly-ash depends on pressure, while other factors contribute 21 percent.

Chapter 6

Summary And Recommendations

This study investigated the relationship between Fly-Ash production at SonySugar Factory, and the chimney temperature and pressure variations in Migori County, Kenya.The study revealed a heterogeneous quantity of fly-ash which was an indication that the quantities (i.e. Concentration of fly-ash,temperature and pressure) were not in harmony,that is the concentration of fly-ash according to both temperature and pressure regulations was not close to the given range. This was apparent in the graph of temperature and pressure of this work and also in quantities of the pollutant, fly-ash from the tables. The high concentration could also be due to the fact that the factory equipment were obsolete and this could be the major source of this menace of high production of the pollutant.

We thus recommend that for proper understanding of the fly-as pollution, the three variables must operate in harmony. Also, since our study engaged the use of finite element equations and use of MATHEMATICA and MaTLaB to compute our matrices and construct graphs,other models be used to investigate the fly-ash pollution . More research should also be done covering other companies.

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