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ON THE USE OF AN AGGREGATED MATRIX MODEL TO DESCRIBE POPULATION DYNAMICS OF MIGRATING INDIVIDUALS

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ABSTRACT

In this paper, the age-structured population matrix, also known as Leslie matrix, has been extended to model the dynamics of migrating individuals. Aggregation techniques have been used to estimate the parameters of the model, and an example of a migration model is proposed.

Keywords: *Aggregation, Leslie Matrix, Migration, population.*

INTRODUCTION

The study of population dynamics is an area that has been of much interest to many scientists, especially biologists and mathematicians. These studies are done with a major intention of predicting the future characteristics of a given biological population. Usually, either the past or the present population structures are known.

Ecologists may be interested in controlling the population of destructive insects and pests, in as much as the farmer would need to optimize their resources so as to get maximum harvests or output. Hence the need to know the dynamics of a given population. This is achieved through modelling of the survivals and reproduction rates of these populations.

Matrix population models have been widely used, especially in situations where the population is classified by age (Leslie1945, 1948). Leslie divided his population of females into age-groups, and expressed the basic age-specific projection equations in matrix form. This model was however found to be inadequate, when considering species with individuals grouped by stages, (Richter, 1990). An Extended matrix was therefore developed to incorporate each of the developmental stages.

In developing his matrix, Leslie made an assumption that fertility and survival rates remain constant over a period of time. He also assumed that there was no migration of individuals.

In this study, we have relaxed some of the assumptions made in Leslie's work, and in particular, we have considered the case where there is migration of individuals. Fertility and survival rates of these individuals therefore depend on the respective regions where they are found.

THE PROPOSED MIGRATION MODEL

Let there be $m = 1,2,\ldots,n$ regions, each region having $p = 1,2,\ldots,m_s$ age-classes.

We denote $x_{ij}(t)$ = expected number of individuals in age class i of region j at time t.

These equations will give rise to the following notation,

$$X(t) = \left(x_{1,}x_{2,}..,x_p\right)^T(t) \tag{1}$$

and

$$s_i(t) = \sum x_{ij}(t) \tag{2}$$

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We now suppose again that the migration rates between different regions by individuals of the same age class i are dependent on the vector s. These migration rates form a regular mxm stochastic matrix $P_i(s)$, for every given value of s.

Leslie matrix parameters come into play such that,

S_{ij} -The portion of individuals who survive from age class i of region j to age class $i+1$ of the same region.
And
F_{ij} -Expected number of offspring per individual of age class i of a region j.

The generalised Leslie matrix is now given as follows:

$$L = \begin{bmatrix} F_1 & F_2 & \cdots & F_{p-1} & F_p \\ S_1 & 0 & \cdots & 0 & 0 \\ 0 & S_2 & \cdots & 0 & 0 \\ 0 & S_2 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & S_{p-1} & 0 \end{bmatrix} \tag{3}$$

And for the migration model, we propose that this matrix gives rise to the following aggregated model,

$$X(t+1) = LP^k\left[(sX)(t)\right] \tag{4}$$

Where L is assumed to be non-linear, and density dependent, and the methods for arriving at the model, are known as aggregation.

AGGREGATION TECHNIQUES FOR DISCRETE MODELS

We divide our population, whose evolution is described in discrete time, into p groups, and each of the groups is divided into several sub-groups. The state of the population at time t is represented by the vector

$$X(t) = \left(x_{1,}x_{2,..},x_p\right)^T(t) \tag{5}$$

The total number of individuals in every group will be given as

$$s_i(t) = \sum x_{ij}(t) \tag{6}$$

So if **1** was the row vector with all entries equal to **1**, then $s_i(t) = 1\mathrm{x}_i(t)$, an also if U is the matrix defined as

U = diag{ $1\mathrm{N}_{1,..}1\mathrm{N}_p$ }, then we obtain

$$s(t) = \left(s_{1,}s_{2,..},s_p\right)^T(t) = \left(1\mathrm{x}_{1,1}x_{2,..},1\mathrm{x}_p\right)^T(t) = UX(t) \tag{7}$$

In this population, it is possible to distinguish two different time scales, the fast and the slow dynamics, with the fast dynamic being non-linear, and dependent on the total number of individuals. We make use of the time unit corresponding to the slow process to determine our discrete process. This is represented by a non-negative matrix M of dimensions nxn. A more generalised form of the model would then be given as

$$s(t+1) = MP^k\left(UX(t)\right)X(t) = MP^k\left[(sX)(t)\right] \tag{8}$$

Where the fast process is given by the k-th power of the matrix P and k is very large. The aggregated model then arises form the generalised model.

The aggregated model describes the dynamics of the fast population dynamics vis a vis the total population $s(t)$.

This is obtained as $s(t+1) = UX(t+1) = UMP^k\left[(sX)(t)\right]$

It also is possible to show that this model converges to an equilibrium state.

CONCLUSIONS

In this paper, we have attempted to model an age structured population in a multi-regional context, by differentiating between a fast and a slow time scale. It is possible to reduce a complex model to a non-linear matrix equation and have its coefficients reflect the situation of migration. We have effectively shown this by extending the study of a normal Leslie matrix to an extended one. This is a prelude to other studies which would take into account both the fast and slow dynamics, and come up with a more generalised aggregated model.

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